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A ROBUST COMPARISON POWERS OF FOUR MULTIVARIATE ANALYSIS OF VARIANCE TESTS

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ABSTRACT: *This paper compared the powers of four test statistics of Multivariate Analysis of Variance and test statistics include Lawley-Hotelling ,Pillai's trace, Roy's largest root and Wilks' lambda. The R Statistics was employed to simulate the data used to compare the four test statistics under the Multivariate Gamma and Multivariate Normal distributions. The sample sizes utilized were 10, 20, 30, 40, 100, 200, 300, 400, 600, 700, 800 and 1000); Number of variables (p = 2, 3, and 4) equal and unequal samples and variance co-variance matrix were also used. The comparison were done at two levels of significance (α = 0.01 and 0.05) using power of the test. The results obtained indicated that the Roy's largest Root test statistic is better than all other test statistic considered when p = g = 2 because it has the least powers. The result of the analysis further showed that when p = g = 3 and p = g = 4 the Wilks' lambda proved better than all other test statistics both for small and large sample sizes. The results equally showed that when the data are multivariate normal and Gamma with g =2 and p = 2 the power of the four test statistics from best to least is Roy's largest root, followed Lawley's trace = Pillae's trace and the least is Wilks' Lambda at significant levels of 0.01 and 0.05 for equal and unequal samples. More so, when data are multivariate normal as well as gamma and p= g = 3 and p=g=4, the power of the four statistics ranked from best to least is Wilks' Lambda , Pillae's trace = Lawley's trace and Roy's largest root respectively . From the findings, it is evidently obvious that Roy's largest root should be applied when p=g=2 while Wilks' Lambda be used when p=g=3 and p=g=4. This study will help researchers to plan studies with controlled probabilities of detecting a meaningful effect thereby giving conclusive results with maximum efficiency.*

KEYWORDS: Power of test, MANOVA Statistic Tests, Stimulated data, Multivariate Gamma distribution, Multivariate Normal.

INTRODUCTION

Multivariate analysis of variance (MANOVA) is a statistical tool in which the single response variable is replaced by several variables. These variables might be substantively different from each other (height, weight); or they might be the same substantive item measured at a number of different times. . The multivariate analysis of variance (MANOVA) technique is similar to univariate analysis of variance and main distinguishing feature is that multiple dependent variables are used for MANOVA while a single variable is used for univariate (Rencher, 2015).

MANOVA has three basic assumptions that are fundamental to the statistical theory which include Independence, Multivariate normality and equality of variance-covariance matrices. A statistical test procedure is said to be insensitive if departures from these assumptions do not greatly affect the significance level or power of the test. But violations in assumptions of multivariate normality and homogeneity of covariance may affect the power of the test and type I error rate of multivariate analysis of variance (Finch, 2005; and Fouladi and Yockey, 2002). This research work will compare powers of four test statistics of MANOVA with objectives of identifying the robustness of the powers of the test statistics of multivariate analysis of variance , determining if datasets will require specific test statistic and ascertaining the test statistic that is better than others.

LITERATURE REVIEW

Cohen, J. (1973) stressed the importance for researchers to carry out more research on power calculations. He stated that power analysis is a powerful, in fact the only rational, guide to planning the relevant details of the research. In his book, Statistical Power Analysis for the Behavioral Sciences, Cohen developed several power calculations by providing readily assessable tables as well explained in details the small, medium and large effect sizes. Prior to this time, there have been substantial contemplations on the estimation and comparison of power in uni-variate analysis of variance (Brewer, J. K. (1972); Tversky&Kahneman, 1971).Brewer, J. K. (1972) stated that educational researchers all over the globe have accepted that the power of a statistical test is important and should be substantial. However, what they have not universally accepted or known is that power can compared for every standard statistical test .He also reiterated that many studies in educational and psychological research lack power calculations in terms of comparison, estimation, etc., and this neglect of power by researchers has protracted and lingered for several years.

Timm, N. H. (1975) and Morrison, D. F. (1967) provided instances to illustrate the comparison of power for two-group MANOVA case, for Hotelling's T^2. However, none of them relates estimation or comparison of power in MANOVA to Cohen's (1973) notion of effect size in univariate. Additionally, they didn’t provide any indication of how much power a researcher would have with small or moderate sample sizes for the kind of effect sizes one frequently encounters in social science research.

Ito, K. (1962). In his paper attempted to compare the powers of Hotelling's T test and Wilks's likelihood ratio W test for moderately large samples .He concluded that the Hotelling's T test and Wilks's likelihood ratio test tend to be identical in distribution as n approaches infinity. He argued that this partly provides clarity on the question in terms of choice, when choosing which of them to use in real life applications or practice. He further stressed that as long as n is very large from the standpoint of power there is no advantage of Hotelling's T test over Wilks's likelihood ratio test.

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Sheehan-Holt, J. K. (1998) carried out study which indicated that the performance of the MANOVA test statistics compared in his research work can be significantly compromised when violations are introduced in the assumption of equality of covariance matrices.

Fouladi &Yockey(2002) emphasized that the results of all these prior studies all point to one direction and no one of the four MANOVA test statistics is clearly optimal under all situations when the assumptions are violated.

MATERIAL AND METHODS

Method of data collection

The sample sizes utilized were 10, 20, 30, 40, 100, 200, 300, 400, 600, 700, 800 and 1000); number of variables (p = 2, 3, and 4); equal and unequal sample, and variance co-variance matrix. The comparisons were done at two levels of significance (α = 0.01 and 0.05) using power of the test.

METHODS OF DATA ANALYSIS

Simulation Using R Statistics

This study made use of a simulation using R statistics to compare the power of the four test multivariate analysis of variance (MANOVA) test statistics.
This simulation was conducted in each of the two different scenarios

- When the null hypothesis (Ho)is true
- When the dataset is normal or not.
- When the equality of variance co-variance matrix hold or not.

Additionally, three factors were varied in the simulation: They include

- The number of groups (g)
- The number of variables (p)
- Significance levels (α).

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Test Statistic of MANOVA

Wilks' lambda

$$\Lambda = \frac{|E|}{|H+E|}$$

$$a = N - g - \frac{p-g+2}{2}$$

$$b = \begin{cases} \sqrt{\dfrac{p^2(g-1)-4}{p^2+(g-1)^2-5}}; & if\ p^2+(g-1)-5 > 0 \\ 1 & if\ p^2+(g-1)-5 \le 0 \end{cases}$$

Pillai Trace

$$V = trace\left(\boldsymbol{H}(\boldsymbol{H}+\boldsymbol{E})^{-1}\right) = \sum_{i=1}^{s}\left(\frac{\lambda_i}{1+\lambda_i}\right)$$

here

$$F = \left(\frac{2u+s+1}{2t+s+1}\right)\left(\frac{V}{s-V}\right) \sim F_{s(2t+s+1),s(2u+s+1)}$$

and

$$U = \frac{N-g-p-1}{2}$$

Roy's largest Root;

$$\theta = \left(\boldsymbol{HE}^{-1}\right) = \frac{\lambda_1}{1+\lambda_1}$$

su+1)

$$F = \left(\frac{2U+2}{2t+2}\right)\phi_{max} \sim F_{(2t+2),(2u+2)}$$

Where $s = \min(p, g-1)$

$$\phi_i = \frac{\theta_i}{1-\theta_i} = \frac{1-\lambda_i}{\lambda_i}$$

$$\lambda_i = 1-\theta_i$$

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RESULTS OF DATA ANALYSIS

Table 1: Multivariate Normal When Variance Co-variance Matrix Are Equal

Table 1: Multivariate Normal when variance co-variance matrix are equal

			Power of the Test for Multivariate Normal							
			α=0.01				α=0.05			
			Wilk	Lawley	Pillai	Roy	Wilk	Lawley	Pillai	Roy
$\Sigma_1 = \Sigma_2$ and $p = g = 2$	$n_1 = n_2$	10,10	0.014	0.013	0.014	0.013	0.058	0.057	0.058	0.058
		100,100	0.104	0.103	0.104	0.101	0.438	0.436	0.438	0.422
		1000,1000	0.459	0.459	0.459	0.455	0.248	0.248	0.248	0.248
	$n_1 \neq n_2$	10,20	0.053	0.051	0.053	0.047	0.130	0.127	0.130	0.123
		100,200	0.065	0.064	0.065	0.064	0.853	0.851	0.853	0.825
		600,1000	0.264	0.263	0.264	0.262	0.567	0.567	0.567	0.564

Table 2: Multivariate Normal When Variance Covariance Matrix Are Not Equal

Table 2: Multivariate Normal when variance co-variance matrix are not equal

			Power of the Test for Multivariate Normal							
			α=0.01				α=0.05			
			Wilk	Lawley	Pillai	Roy	Wilk	Lawley	Pillai	Roy
$\Sigma_1 \neq \Sigma_2$ and $p = g = 2$	$n_1 = n_2$	10,10	0.018	0.017	0.018	0.018	0.175	0.167	0.175	0.152
		100,100	0.133	0.132	0.133	0.128	0.058	0.058	0.058	0.058
		1000,1000	0.087	0.087	0.087	0.087	0.503	0.503	0.503	0.501
	$n_1 \neq n_2$	10,20	0.042	0.040	0.042	0.039	0.068	0.067	0.068	0.068
		100,200	0.053	0.053	0.053	0.053	0.313	0.312	0.313	0.308
		600,1000	0.148	0.148	0.148	0.148	0.134	0.134	0.134	0.134

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Table 3: **Multivariate Gamma When Variance Covariance Matrix Are Equal**

Table 3: Multivariate gamma when variance co-variance matrix are equal

			Power of the Test for Multivariate Gamma							
			α=0.01				α=0.05			
			Wilk	**Lawley**	**Pillai**	**Roy**	**Wilk**	**Lawley**	**Pillai**	**Roy**
$\Sigma_1 = \Sigma_2$ and p = g = 2	$n_1 = n_2$	10,10	0.025	0.024	0.025	0.024	0.065	0.065	0.066	0.065
		100,100	0.320	0.318	0.320	0.298	0.117	0.116	0.117	0.116
		1000,1000	0.569	0.569	0.569	0.564	0.816	0.816	0.816	0.812
	$n_1 \neq n_2$	10,20	0.010	0.010	0.010	0.010	0.080	0.078	0.080	0.079
		100,200	0.111	0.111	0.111	0.109	0.387	0.386	0.387	0.379
		600,1000	0.237	0.236	0.237	0.235	0.840	0.839	0.840	0.834

Table 4: **Multivariate Gamma When Variance Co-variance Matrix Are Not Equal**

Table 4: Multivariate gamma when variance co –variance matrix are not equal

			Power of the Test for Multivariate Gamma							
			α=0.01				α=0.05			
			Wilk	**Lawley**	**Pillai**	**Roy**	**Wilk**	**Lawley**	**Pillai**	**Roy**
$\Sigma_1 \neq \Sigma_2$ and p = g = 2	$n_1 = n_2$	10,10	0.043	0.040	0.043	0.037	0.072	0.070	0.072	0.071
		100,100	0.111	0.111	0.111	0.108	0.112	0.111	0.112	0.111
		1000,1000	0.891	0.891	0.891	0.885	0.975	0.975	0.975	0.973
	$n_1 \neq n_2$	10,20	0.161	0.154	0.161	0.116	0.086	0.084	0.086	0.084
		100,200	0.101	0.101	0.101	0.099	0.121	0.121	0.121	0.120
		600,1000	0.394	0.394	0.394	0.390	0.840	0.840	0.840	0.835

Table 5: **Multivariate Normal When Variance Covariance Matrix Are Equal**

Table 5: Multivariate normal when variance co-variance matrix are equal

			Power of the Test for Multivariate Normal							
			α=0.01				α=0.05			
			Wilk	**Lawley**	**Pillai**	**Roy**	**Wilk**	**Lawley**	**Pillai**	**Roy**
$\Sigma_1 = \Sigma_2$ and p = g = 3	$n_1 = n_2$	10,10,10	0.013	0.016	0.017	0.051	0.054	0.060	0.061	0.093
		100,100,100	0.021	0.028	0.028	0.152	0.081	0.100	0.100	0.325
		1000,1000,1000	0.056	0.094	0.093	0.651	0.134	0.185	0.185	0.578
	$n_1 \neq n_2$	10,20,30	0.012	0.017	0.017	0.062	0.085	0.152	0.154	0.434
		100,200,300	0.017	0.026	0.026	0.084	0.204	0.407	0.399	0.959
		600,800,1000	0.037	0.067	0.068	0.267	0.109	0.160	0.160	0.392

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Multivariate Normal When Variance Covariance Matrix Are Not Equal

Table 6: Multivariate normal when variance co-variance matrix are not equal

			Power of the Test for Multivariate Normal							
			α=0.01				α=0.05			
			Wilk	Lawley	Pillai	Roy	Wilk	Lawley	Pillai	Roy
$\Sigma_1 \neq \Sigma_2$ and $p = g = 3$	$n_1 = n_2$	10,10,10	0.013	0.018	0.018	0.060	0.076	0.104	0.102	0.305
		100,100,100	0.020	0.026	0.026	0.141	0.060	0.065	0.065	0.126
		1000,1000,1000	0.016	0.020	0.020	0.063	0.059	0.063	0.063	0.122
	$n_1 \neq n_2$	10,20,30	0.013	0.018	0.018	0.071	0.066	0.093	0.092	0.293
		100,200,300	0.013	0.017	0.017	0.063	0.072	0.099	0.099	0.329
		600,800,1000	0.013	0.016	0.016	0.059	0.067	0.080	0.080	0.213

Table 7

Multivariate Gamma When Variance Covariance Matrix Are Equal

Table 7: Multivariate gamma when variance co –variance matrix are equal

			Power of the Test for Multivariate Gamma							
			α=0.01				α=0.05			
			Wilk	Lawley	Pillai	Roy	Wilk	Lawley	Pillai	Roy
$\Sigma_1 = \Sigma_2$ and $p = g = 3$	$n_1 = n_2$	10,10,10	0.024	0.048	0.044	0.194	0.057	0.063	0.065	0.098
		100,100,100	0.035	0.055	0.054	0.375	0.109	0.145	0.146	0.345
		1000,1000,1000	0.232	0.416	0.411	0.994	0.140	0.196	0.196	0.738
	$n_1 \neq n_2$	10,20,30	0.014	0.021	0.022	0.057	0.062	0.081	0.081	0.218
		100,200,300	0.017	0.027	0.027	0.157	0.120	0.212	0.212	0.595
		600,800,1000	0.234	0.500	0.492	0.998	0.219	0.372	0.370	0.958

Table 8

Multivariate Gamma When Variance Covariance Matrix Are Not Equal

Table 8: Multivariate gamma when variance co-variance matrix are not equal

			Power of the Test for Multivariate Gamma							
			α=0.01				α=0.05			
			Wilk	Lawley	Pillai	Roy	Wilk	Lawley	Pillai	Roy
$\Sigma_1 \neq \Sigma_2$ and $p = g = 3$	$n_1 = n_2$	10,10,10	0.016	0.024	0.026	0.071	0.078	0.106	0.111	0.259
		100,100,100	0.020	0.027	0.027	0.114	0.077	0.093	0.093	0.300
		1000,1000,1000	0.186	0.338	0.335	0.984	0.219	0..324	0.323	0.925
	$n_1 \neq n_2$	10,20,30	0.014	0.022	0.022	0.092	0.092	0.167	0.170	0.449
		100,200,300	0.022	0.039	0.038	0.272	0.107	0.181	0.180	0.624
		600,800,1000	0.099	0.218	0.216	0.926	0.356	0.587	0.583	0.996

Table 9: **Multivariate Gamma When Variance Covariance Matrix Are Not Equal**

Table 9: Multivariate gamma when variance co –variance matrix are not equal

			Power of the Test for Multivariate Normal							
			α=0.01				α=0.05			
			Wilk	**Lawle**	**Pillai**	**Roy**	**Wilk**	**Lawle**	**Pillai**	**Roy**
$\Sigma_1 \neq \Sigma_2$ and $p = g = 4$	$n_1 = n_2$	10,10,10,10	0.011	0.013	0.014	0.073	0.052	0.060	0.061	0.163
		100,100,100,100	0.015	0.022	0.022	0.230	0.073	0.103	0.103	0.663
		1000,1000,1000,1000	0.016	0.024	0.024	0.433	0.145	0.279	0.277	0.999
	$n_1 \neq n_2$	10,20,30,40	0.011	0.015	0.015	0.078	0.054	0.073	0.074	0.392
		100,200,300,400	0.014	0.028	0.028	0.351	0.059	0.085	0.056	0.434
		600,700,800,1000	0.029	0.079	0.078	0.970	0.102	0.200	0.199	0.987

Table 10 : **Multivariate Normal When Variance Covariance Matrix Are Not Equal**

Table 10: Multivariate normal when variance co-variance matrix are not equal

			Power of the Test for Multivariate Normal							
			α=0.01				α=0.05			
			Wilk	**Lawle**	**Pillai**	**Roy**	**Wilk**	**Lawle**	**Pillai**	**Roy**
$\Sigma_1 \neq \Sigma_2$ and $p = g = 4$	$n_1 = n_2$	10,10,10,10	0.013	0.023	0.022	0.231	0.057	0.078	0.077	0.401
		100,100,100,100	0.016	0.026	0.026	0.479	0.066	0.087	0.087	0.591
		1000,1000,1000,1000	0.021	0.038	0.038	0.714	0.078	0.113	0.113	0.774
	$n_1 \neq n_2$	10,20,30,40	0.012	0.019	0.019	0.269	0.058	0.099	0.098	0.597
		100,200,300,400	0.012	0.017	0.017	0.164	0.055	0.069	0.069	0.245
		600,700,800,1000	0.019	0.039	0.039	0.519	0.068	0.096	0.096	0.637

Table 11: **Multivariate Gamma When Variance Covariance Matrix Are Equal**

Table 11: Multivariate gamma when variance co –variance matrix are equal

			Power of the Test for Multivariate Gamma							
			α=0.01				α=0.05			
			Wilk	**Lawl**	**Pillai**	**Roy**	**Wilk**	**Lawle**	**Pillai**	**Roy**
$\Sigma_1 = \Sigma_2$ and $p = g = 4$	$n_1 = n_2$	10,10,10,10	0.011	0.015	0.015	0.105	0.056	0.075	0.076	0.369
		100,100,100,100	0.013	0.018	0.018	0.142	0.063	0.079	0.079	0.382
		1000,1000,1000,1000	0.017	0.027	0.027	0.481	0.059	0.070	0.070	0.301
	$n_1 \neq n_2$	10,20,30,40	0.012	0.022	0.022	0.267	0.053	0.069	0.069	0.368
		100,200,300,400	0.013	0.020	0.020	0.310	0.061	0.093	0.093	0.487
		600,700,800,1000	0.013	0.018	0.018	0.226	0.066	0.093	0.093	0.465

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Multivariate Gamma When Variance Covariance Matrix Are Not Equal

Table 12: Multivariate gamma when variance co –variance matrix are not equal

			Power of the Test for Multivariate Gamma							
			α=0.01				α=0.05			
			Wilk	Lawle	Pillai	Roy	Wilk	Lawle	Pillai	Roy
$\Sigma_1 \neq \Sigma_2$ and p = g = 4	$n_1 = n_2$	10,10,10,10	0.011	0.015	0.015	0.078	0.059	0.085	0.087	0.394
		100,100,100,100	0.014	0.019	0.019	0.187	0.079	0.119	0.118	0.813
		1000,1000,1000,1000	0.016	0.025	0.025	0.376	0.070	0.092	0.092	0.645
	$n_1 \neq n_2$	10,20,30,40	0.011	0.017	0.017	0.160	0.055	0.082	0.082	0.467
		100,200,300,400	0.014	0.027	0.027	0.379	0.069	0.129	0.129	0.883
		600,700,800,1000	0.012	0.015	0.015	0.150	0.067	0.095	0.095	0.534

CONCLUSION

This study examined the powers of four test statistics of Multivariate Analysis of Variance. The four compared test statistics include Lawley-Hotelling trace, Pillai's trace, Roy's largest Root and Wilks' lambda. The data was simulated to compare the four test statistics under two different distributions (multivariate Gamma and multivariate normal), sample sizes (10, 20, 30, 40, 100, 200, 300, 400, 600, 700, 800 and 1000), number of variables (p = 2, 3, 4), equal and unequal sample and variance co-variance matrix. The comparisons were done at two levels of significance (α = 0.01 and 0.05) using power of the test.

From all the result of the analysis, it is clear that when the levels of significance (α) increases, the power of the four test statistics also increases. Further indicated that when the assumption of equality of variance co–variance matrix is altered or violated, it affects the power of the four test statistics, as long as p and g = 2 and the sample sizes are very small. Also showed that when p = g = 2 and p = g = 3, the power of the four test statistics are significantly affected when variance co-variance matrix are unequal and data are multivariate gamma. More so, when p and g = 4 unequal variance co-variance matrix and multivariate gamma does not have any effect on the power of the four test statistics. It is obviously shown when p and g = 2 and null hypothesis is true, Roy's largest root is better than the three other test statistics, while Wilks' Lambda is better than others when p = g = 3 and p = g = 4.

This study will help researchers to plan studies with controlled probabilities of detecting a meaningful effect thereby giving conclusive results with maximum efficiency. More research is also required to be done featuring more sample sizes, other multivariate tests statistics and other criteria.

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