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Approximate Solution of the Fractional Order Mathematical Model on the Transmission Dynamics on The Co-Infection of COVID-19 and Monkeypox Using the Laplace-Adomian Decomposition Method

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ABSTRACT: *A fractional order compartmental model on the transmission dynamics of the co-infection of COVID-19 and Monkeypox is presented. The approximate solutions of the fractional order model are obtained using the Laplace-Adomian Decomposition method in the form of an infinite series which was shown to converge to the exact value. Using the MATLAB fmincon algorithm, we carried out a data fitting analysis using real life COVID-19 and Monkeypox data so as to obtain estimates for some of the key parameters used in the formulation of model. The results of our analysis showed that an increase in the effective treatment capacity in the human population will significantly reduce the burden of these diseases in the human population.*

KEYWORDS: Approximate solution, fractional order mathematical model, transmission dynamics, the co-infection of COVID-19, monkeypox, laplace-adomian decomposition method

INTRODUCTION

COVID-19, a transmissible illness instigated by the SARS-CoV-2 virus, emerged initially in Wuhan, China, in December 2019. It swiftly propagated globally, culminating in the COVID-19 pandemic. [1] COVID-19 spreads when contagious particles are inhaled or make contact with the eyes, nose, or mouth. The highest risk occurs in close proximity, but tiny airborne particles carrying the virus can linger in the air and travel farther, especially indoors. Transmission can also happen through touching surfaces or objects contaminated by the virus and subsequently touching the eyes, nose, or mouth. Contagiousness can last for up to 20 days, and individuals can transmit the virus even without displaying symptoms. [2] In community and healthcare environments, the utilization of face masks and hand sanitizers serves as a means of source containment to minimize the spread of the virus and as a precautionary measure to avert infection. Appropriately utilized face masks not only restrict the dissemination of respiratory droplets and aerosols from individuals who are infected but also safeguard uninfected individuals from contracting the virus. [3]

Mpox (previously identified as monkeypox) is a contagious viral illness that can manifest in humans and certain other creatures.[4] Manifestations comprise a skin eruption leading to blistering followed by crust formation, elevated body temperature, and enlarged lymph glands. Typically, the ailment is mild, and the majority of affected individuals will recuperate spontaneously within a couple of weeks, even without medical intervention. [5] The disease is caused by the monkeypox virus, a zoonotic virus in the genus *Orthopoxvirus*. The variola virus, the causative agent of the disease smallpox, is also in this genus. [4] There isn't a particular remedy for the illness, although antiviral drugs like tecovirimat have been sanctioned for severe mpox treatment. [6] A Cochrane analysis in 2023 revealed no concluded randomized controlled studies examining therapies for Mpox. Instead, it highlighted non-randomized controlled trials assessing the safety of Mpox treatments, indicating no major risks from tecovirimat and limited evidence suggesting brincidofovir could induce mild liver damage. [7]

Mathematical model serves as an invaluable tool in studying and analyzing the transmission dynamics of contagious diseases within a population, thus several models have been developed by various authors in a bid to make recommendations to health care personnel so as to reduce the disease burdens of infectious diseases within any population. For instance Atokolo et. al. [8] introduced a fractional order sterile insect technology (SIT) model to combat Zika virus transmission, employing the Laplace–Adomian decomposition method (LADM) to derive an analytical solution. They demonstrated that the fractional model offers greater flexibility, allowing for varied responses by adjusting the fractional order. Their work contributes to the literature by showcasing the applicability of LADM in solving SIT models, a novel approach in the field. Atokolo et al.[9] examined the impact of parameter values (θ, φ, h, and γ) on reducing the basic reproduction number ($R0$) of COVID-19, suggesting that adjusting these parameters could lead to the eventual elimination of the disease from the population. Their numerical simulations indicated that proper adherence to control measures such as social distancing, hand hygiene, and coughing etiquette could contribute to the eradication of the disease over time. Additionally, increasing rates of quarantine and isolation for suspected and confirmed cases were found to be effective in reducing the spread of the pandemic.

Model Formulation

A deterministic compartmental model on the transmission dynamics of the co-infection of COVID-19 and Monkeypox disease is been proposed. The model comprises of two populations, which are the human population and the rodent population as the reservoirs.

We further sub-divide the human population into ten compartments, namely, Susceptible humans $S_h(t)$, Exposed COVID-19 humans $E_c(t)$, Exposed monkey pox humans $E_m(t)$, Infected COVID-19 humans $I_c(t)$, Infected monkey pox humans $I_m(t)$, quarantined COVID-19 humans $Q_c(t)$, Quarantined monkey pox humans $Q_m(t)$, Treated class of humans $T(t)$, and Recovered class of humans $R(t)$. The rodent population is sub-divided into three compartments, namely, susceptible rodents $S_r(t)$, Exposed rodents $E_r(t)$, and infected rodents $I_r(t)$.

The recruitment rate of individuals into the susceptible population is denoted as π_h. β_c represents the effective contact rate, indicating the likelihood of humans contracting COVID-19 per contact with an infected person. Similarly, β_1 signifies the effective contact rate for humans acquiring Monkeypox through contact with an infected rodent. β_m denotes the effective contact rate for humans contracting Monkeypox from an infected individual, while β_r stands for the effective contact rate with the probability of rodents getting infected with Monkeypox through contact with an infected rodent.

The progression rate of individuals exposed to COVID-19 into the infected COVID-19 Population is θ_1. θ_2 is the progression rate of individuals exposed to Monkeypox into the infected Monkeypox compartment. ϕ_1 and ϕ_2 are the rates at which individuals exposed to COVID-19 and Monkeypox are moved to the quarantined COVID-19 and quarantined monkeypox center respectively. The natural death rate of the human population is μ_h. The disease induced death rate of the infected and quarantined COVID-19 individuals is δ_1. The disease induced death rate of the infected and quarantined Monkeypox individuals is δ_2. The disease induced death rate of the individuals in the co-infection compartment is δ_3. The disease induced death rate of the individuals in the treated class is δ_4. ψ is the modification parameter that accounts for reduced disease induced death rate in the treatment class.

The force of infection for the human-to-human COVID-19 transmission is given as

$$\lambda_1 = \frac{\beta_c \left(I_c + I_{cm}\right)}{N_h}.$$

The force of infection for the human-to-human with the rodent-to-human Monkeypox transmission is given as

$$\lambda_2 = \frac{\beta_1 I_r + \beta_m \left(I_m + I_{cm}\right)}{N_h}.$$

The force of infection for the rodents-to-rodents disease transmission is given as

$$\lambda_3 = \frac{\beta_r I_r}{N_r}.$$

After medical diagnosis, the undetected proportion of the quarantined population which do not show clinical symptoms of the diseases, namely, COVID-19 and Monkeypox are returned to the susceptible population at the rate of ω_1 and ω_2 respectively. γ_4 and γ_5 are the rates at which individuals that show clinical symptoms to COVID-19 and Monkeypox are moved to the treated class respectively. The incidence of individuals contracting both COVID-19 and Monkeypox infections is determined by the parameter τ_1. τ_2 is the rate at which individuals infected with Monkeypox only becomes also infected with COVID-19. γ_1, γ_2 and γ_3 are the rates at which infected COVID-19 individuals, infected Monkeypox individuals, and co-infected COVID-19 and Monkeypox individuals respectively are been moved to the treated class. ε is the rate at which individuals in the treated class respond positively to treatment and progress to the recovery class. α is the waning rate of recovered COVID-19 individuals (i.e. the rate at which recovered COVID-19 individuals becomes susceptible to it again).

The recruitment rate of rodents into the rodents population is given by π_r. The progression rate of rodents exposed to Monkeypox into the infected class is denoted by θ_3. The natural death rate and the disease induced death rates of the rodents population are μ_r and δ_r respectively.

The total human population is given as

$$N_h(t) = S_h(t) + E_c(t) + E_m(t) + Q_c(t) + Q_m(t) + I_c(t) + I_m(t) + I_{cm}(t) + T(t) + R(t)$$

The total rodents population is given as

$$N_r(t) = S_r(t) + E_r(t) + I_r(t)$$

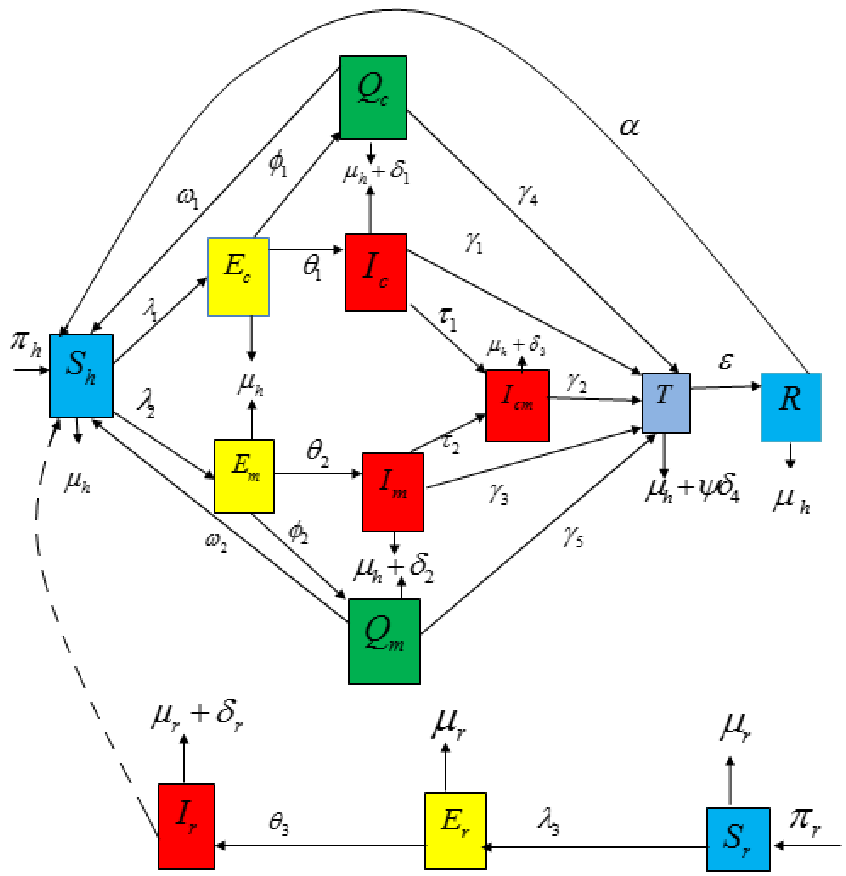


Figure 1: Model Flow Diagram.

Model Assumptions

The assumptions used in the formulation of the model are:

- (i.) No transmission of the infection(s) occurs vertically from mother to unborn child. [10]
- (ii.) There is homogeneous mixing which implies that all susceptible individuals equally risk infection upon contact with those who are infectious.
- (iii.) Disease-induced fatalities occur exclusively in the infectious compartments, with a consistent natural death rate across all compartments.
- (iv.) The rodent population can spread only Monkeypox and not COVID-19. [11]
- (v.) Individuals who have recovered from COVID-19 can become susceptible to the virus once more. [12]

Parameters	Description
π_h	Human recruitment rate
π_r	Rodents recruitment rate
β_c	Contact rate of susceptible and Infected COVID-19 humans
β_m	Contact rate of susceptible humans and Infected Monkeypox humans
β_1	Contact rate of susceptible humans and Infected Rodents
β_r	Contact rate of susceptible rodents and Infected Rodents
λ_1	Force of Infection for COVID-19 Infection
λ_2	Force of Infection for Monkeypox Infection amongst humans
λ_3	Force of Infection for Monkeypox Infection amongst Rodents
ω_1	Progression rate from Quarantine COVID-19 individual to susceptible humans
ω_2	Rate of progression from Quarantine monkeypox individual to susceptible humans
ϕ_1	Progression rate from exposed COVID-19 individual to Quarantine COVID-19 humans
ϕ_2	Progression rate from exposed monkeypox individual to Quarantine monkeypox humans
θ_1	Progression rate from exposed COVID-19 individual to Infected COVID-19 humans
θ_2	Progression rate from exposed monkeypox individual to Infected monkeypox humans
θ_3	Progression rate from exposed Monkeys to Infected Rodents
γ_1	Treatment rate of Infected COVID-19 individuals

γ_2	Treatment rate of Co-Infected COVID-19 and monkeypox individuals
γ_3	Treatment rate of Infected Monkeypox individuals
γ_4	Treatment rate of Quarantined COVID-19 individuals
γ_5	Treatment rate of Quarantined Monkeypox individuals
τ_1	Rate at which infected COVID-19 individuals become infected with Monkeypox
τ_2	Rate at which infected monkeypox individuals become infected with COVID-19
ε	Recovery rate of treated Class
α	Waning rate of COVID-19 individuals
μ_h	Natural death rate of humans
μ_r	Natural death rate of rodents
δ_1	Disease induced death rate of infected and quarantined COVID-19 individuals
δ_2	Disease induced death rate of infected and quarantined Monkeypox individuals
δ_3	Disease induced death rate of co-infected individuals
δ_4	Disease induced death rate of individuals in the treatment class
ψ	Modification parameter addressing reduced death rate in the treatment class
δ_r	Disease induced death rate of humans

Table 1: Description of Parameters

Variable	**Description**
$S_h(t)$	Susceptible humans population
$S_r(t)$	Susceptible rodents population
$E_c(t)$	Exposed COVID-19 population
$E_m(t)$	Exposed human Monkeypox Population
$E_r(t)$	Exposed rodents population to Monkeypox

$Q_c(t)$	Quarantined COVID-19 population
$Q_m(t)$	Quarantined human Monkeypox population
$I_c(t)$	Infected COVID-19 Population
$I_m(t)$	Infected Monkeypox human population
$I_r(t)$	Infected rodents population
$I_{cm}(t)$	Population of co-infected COVID-19 and Monkeypox humans
$T(t)$	Treated Class of humans
$R(t)$	Recovered Class

Table 2: Description of Variables

Model Equations

In the light of the description of the model above, we obtained the differential equations modeling the co-infection transmission dynamics of COVID-19 and Monkeypox as follows:

$$\left.\begin{aligned}
\frac{dS_h}{dt} &= \pi_h + \alpha R + \omega_1 Q_c + \omega_2 Q_m - \left(\lambda_1 + \lambda_2 + \mu_h\right) S_h, \\
\frac{dE_c}{dt} &= \lambda_1 S_h - \left(\phi_1 + \theta_1 + \mu_h\right) E_c, \\
\frac{dE_m}{dt} &= \lambda_2 S_h - \left(\phi_2 + \theta_2 + \mu_h\right) E_m, \\
\frac{dQ_c}{dt} &= \phi_1 E_c - \left(\omega_1 + \gamma_4 + \delta_1 + \mu_h\right) Q_c, \\
\frac{dQ_m}{dt} &= \phi_2 E_m - \left(\omega_2 + \gamma_5 + \delta_2 + \mu_h\right) Q_m, \\
\frac{dI_c}{dt} &= \theta_1 E_c - \left(\tau_1 + \gamma_1 + \delta_1 + \mu_h\right) I_c, \\
\frac{dI_m}{dt} &= \theta_2 E_m - \left(\tau_2 + \gamma_3 + \delta_2 + \mu_h\right) I_m, \\
\frac{dI_{cm}}{dt} &= \tau_1 I_c + \tau_2 I_m - \left(\gamma_2 + \delta_3 + \mu_h\right) I_{cm}, \\
\frac{dT}{dt} &= \gamma_1 I_c + \gamma_2 I_{cm} + \gamma_3 I_m + \gamma_4 Q_c + \gamma_5 Q_m - \left(\varepsilon + \psi\delta_4 + \mu_h\right) T, \\
\frac{dR}{dt} &= \varepsilon T - \left(\alpha + \mu_h\right) R, \\
\frac{dS_r}{dt} &= \pi_r - \left(\lambda_3 + \mu_r\right) S_r, \\
\frac{dE_r}{dt} &= \lambda_3 S_r - \left(\theta_3 + \mu_r\right) E_r, \\
\frac{dI_r}{dt} &= \theta_3 E_r - \left(\delta_r + \mu_r\right) I_r.
\end{aligned}\right\} \tag{1}$$

Where $\lambda_1 = \frac{\beta_c\left(I_c + I_{cm}\right)}{N_h}, \lambda_2 = \frac{\beta_1 I_r + \beta_m\left(I_m + I_{cm}\right)}{N_h}, \lambda_3 = \frac{\beta_r I_r}{N_r}$ are the forces of infection.

Fractional Order of the COVID-19 – Monkeypox Co-infection Model

The Caputo derivative is measured as a differential operator in our model. We present in this segment some well-known definitions and effects that we shall be using throughout this research.

Definition 1 The Caputo fractional order derivative of a function (f) on the interval [O,T] is defined by:

$$\left[{}^{C}D_0^{\beta} f(t)\right] = \frac{1}{\Gamma(n-\beta)} \int_0^t (t-s)^{n-\beta-1} f^{(n)}(s)ds, \tag{2}$$

Where $n = [\beta]+1$ and $[\beta]$ represents the integer part of β. In particular, for $0 < \beta < 1,$ the Caputo derivative becomes:

$$\left[{}^{C}D_0^{\beta} f(t)\right] = \frac{1}{\Gamma(1-\beta)} \int_0^t \frac{f(s)}{(t-s)^{\beta}} ds, \tag{3}$$

Definition 2 Laplace transform of Caputo derivatives is defined as

$$\mathscr{L}[{}^{C}D^{\beta} q(t)] = S^{\beta} h(S) - \sum_{K=0}^{n} S^{\beta-i-1} y^{k}(0), \quad n-1 < \beta < n,\ n \in N, \tag{4}$$

For arbitrary $c_i \in R, i = 0,1,2,...n-1,\ n = [\beta]+1$ and $[\beta]$ represents the non-integer part of β .

Lemma 1. The following results hold for fractional differentiation equations

$$I^{\beta}[{}^{c}D^{\beta} h](t) = h(t) + \sum_{i=0}^{n-1} \frac{h^{(t)}(0)}{i!} t^{i}, \tag{5}$$

For arbitrary $\beta > 0, i = 0,1,2,..., n-1,$ where $n = [\beta]+1$ and $[\beta]$ represents the integer part of β

Introducing fractional-order into the model, we now present a new model described by the following Introducing fractional order derivative into the model we present new mathematical model describe by set of fractional difference of order β for $0 < \beta < 1$

$$\left.\begin{aligned}
&D^{\beta}\left(S_h\right)=\pi_h+\alpha R+\omega_1 Q_c+\omega_2 Q_m-\left(\lambda_1+\lambda_2+\mu_h\right)S_h,\\
&D^{\beta}\left(E_c\right)=\lambda_1 S_h-\left(\phi_1+\theta_1+\mu_h\right)E_c,\\
&D^{\beta}\left(E_m\right)=\lambda_2 S_h-\left(\phi_2+\theta_2+\mu_h\right)E_m,\\
&D^{\beta}\left(Q_c\right)=\phi_1 E_c-\left(\omega_1+\gamma_4+\delta_1+\mu_h\right)Q_c,\\
&D^{\beta}\left(Q_m\right)=\phi_2 E_m-\left(\omega_2+\gamma_5+\delta_2+\mu_h\right)Q_m,\\
&D^{\beta}\left(I_c\right)=\theta_1 E_c-\left(\tau_1+\gamma_1+\delta_1+\mu_h\right)I_c,\\
&D^{\beta}\left(I_m\right)=\theta_2 E_m-\left(\tau_2+\gamma_3+\delta_2+\mu_h\right)I_m,\\
&D^{\beta}\left(I_{cm}\right)=\tau_1 I_c+\tau_2 I_m-\left(\gamma_2+\delta_3+\mu_h\right)I_{cm},\\
&D^{\beta}\left(T\right)=\gamma_1 I_c+\gamma_2 I_{cm}+\gamma_3 I_m+\gamma_4 Q_c+\gamma_5 Q_m-\left(\varepsilon+\psi\delta_4+\mu_h\right)T,\\
&D^{\beta}\left(R\right)=\varepsilon T-\left(\alpha+\mu_h\right)R,\\
&D^{\beta}\left(S_r\right)=\pi_r-\left(\lambda_3+\mu_r\right)S_r,\\
&D^{\beta}\left(E_r\right)=\lambda_3 S_r-\left(\theta_3+\mu_r\right)E_r,\\
&D^{\beta}\left(I_r\right)=\theta_3 E_r-\left(\delta_r+\mu_r\right)I_r.
\end{aligned}\right\} \tag{6}$$

The Laplace-Adomian Decomposition Method (LADM) Implementation

We considered the general procedure of this method with the initial conditions. Applying Laplace transforms to both sides of the equation (1), and then we have:

$$\left.\begin{aligned}
&S^{\beta}\mathcal{L}(S_h)-S^{\beta-1}S_h(0)=\mathcal{L}\left[\pi_h+\alpha R+\omega_1 Q_c+\omega_2 Q_m-\left(\lambda_1+\lambda_2+\mu_h\right)S\right]\\
&S^{\beta}\mathcal{L}(E_c)-S^{\beta-1}E_c(0)=\mathcal{L}\left[\lambda_1 S_h-\left(\phi_1+\theta_1+\mu_h\right)E_c\right]\\
&S^{\beta}\mathcal{L}(E_m)-S^{\beta-1}E_m(0)=\mathcal{L}\left[\lambda_2 S_h-\left(\phi_2+\theta_2+\mu_h\right)E_m\right]\\
&S^{\beta}\mathcal{L}(Q_c)-S^{\beta-1}Q_c(0)=\mathcal{L}\left[\phi_1 E_c-\left(\omega_1+\gamma_4+\delta_1+\mu_h\right)Q_c\right]\\
&S^{\beta}\mathcal{L}(Q_m)-S^{\beta-1}Q_m(0)=\mathcal{L}\left[\phi_2 E_m-\left(\omega_2+\gamma_5+\delta_2+\mu_h\right)Q_m\right]\\
&S^{\beta}\mathcal{L}(I_c)-S^{\beta-1}I_c(0)=\mathcal{L}\left[\theta_1 E_c-\left(\tau_1+\gamma_1+\delta_1+\mu_h\right)I_c\right]\\
&S^{\beta}\mathcal{L}(I_m)-S^{\beta-1}I_m(0)=\mathcal{L}\left[\theta_2 E_m-\left(\tau_2+\gamma_3+\delta_2+\mu_h\right)I_m\right]\\
&S^{\beta}\mathcal{L}(I_{cm})-S^{\beta-1}I_{cm}(0)=\mathcal{L}\left[\tau_1 I_c+\tau_2 I_m-\left(\gamma_2+\delta_3+\mu_h\right)I_{cm}\right]\\
&S^{B}L(T)-S^{\beta-1}T(0)=L\left[\gamma_1 I_c+\gamma_2 I_{cm}+\gamma_3 I_m+\gamma_4 Q_c+\gamma_5 Q_m-\left(\varepsilon+\psi\delta_4+\mu_h\right)T\right]\\
&S^{B}\mathcal{L}(R)-S^{\beta-1}R(0)=\mathcal{L}\left[\varepsilon T-\left(\alpha+\mu_h\right)R\right]\\
&S^{B}\mathcal{L}(S_r)-S^{\beta-1}S_r(0)=\mathcal{L}\left[\pi_r-\left(\lambda_3+\mu_r\right)S_r\right]\\
&S^{B}\mathcal{L}(E_r)-S^{\beta-1}E_r(0)=\mathcal{L}\left[\lambda_3 S_r-\left(\theta_3+\mu_r\right)E_r\right]\\
&S^{B}\mathcal{L}(I_r)-S^{\beta-1}I_r(0)=\mathcal{L}\left[\theta_3 E_r-\left(\delta_r+\mu_r\right)I_r\right]
\end{aligned}\right\} \tag{7}$$

With initial conditions

$$S_h(0)=n_1,\ \ E_c(0)=n_2,\ E_m(0)=n_3,\ Q_c(0)=n_4,\ Q_m(0)=n_5,\ I_c(0)=n_6,\ I_m(0)=n_7,$$

$$I_{cm}(0)=n_8,\ T(0)=n_9,\ R(0)=n_{10},\ S_r\left(0\right)=n_{11}, E_r\left(0\right)=n_{12}, I_r\left(0\right)=n_{13}$$

Dividing eqn. (7) by (S^{β}) we have:

$$\left.\begin{aligned}
&\mathcal{L}(S_h)=\frac{n_1}{S}+\frac{1}{S^\beta}\mathcal{L}\left[\pi_h+\alpha R+\omega_1 Q_c+\omega_2 Q_m-\left(\lambda_1+\lambda_2+\mu_h\right)S\right]\\
&\mathcal{L}(E_c)=\frac{n_2}{S}+\frac{1}{S^\beta}\mathcal{L}\left[\lambda_1 S_h-\left(\phi_1+\theta_1+\mu_h\right)E_c\right]\\
&\mathcal{L}(E_m)=\frac{n_3}{S}+\frac{1}{S^\beta}\mathcal{L}\left[\lambda_2 S_h-\left(\phi_2+\theta_2+\mu_h\right)E_m\right]\\
&\mathcal{L}(Q_c)=\frac{n_4}{S}+\frac{1}{S^\beta}\mathcal{L}\left[\phi_1 E_c-\left(\omega_1+\gamma_4+\delta_1+\mu_h\right)Q_c\right]\\
&\mathcal{L}(Q_m)=\frac{n_5}{S}+\frac{1}{S^\beta}\mathcal{L}\left[\phi_2 E_m-\left(\omega_2+\gamma_5+\delta_2+\mu_h\right)Q_m\right]\\
&\mathcal{L}(I_c)=\frac{n_6}{S}+\frac{1}{S^\beta}\mathcal{L}\left[\theta_1 E_c-\left(\tau_1+\gamma_1+\delta_1+\mu_h\right)I_c\right]\\
&\mathcal{L}(I_m)=\frac{n_7}{S}+\frac{1}{S^\beta}\mathcal{L}\left[\theta_2 E_m-\left(\tau_2+\gamma_3+\delta_2+\mu_h\right)I_m\right]\\
&\mathcal{L}(I_{cm})=\frac{n_8}{S}+\frac{1}{S^\beta}\mathcal{L}\left[\tau_1 I_c+\tau_2 I_m-\left(\gamma_2+\delta_3+\mu_h\right)I_{cm}\right]\\
&\mathcal{L}(T)=\frac{n_9}{S}+\frac{1}{S^\beta}\mathcal{L}\left[\gamma_1 I_c+\gamma_2 I_{cm}+\gamma_3 I_m+\gamma_4 Q_c+\gamma_5 Q_m-\left(\varepsilon+\psi\delta_4+\mu_h\right)T\right]\\
&\mathcal{L}(R)=\frac{n_{10}}{S}+\frac{1}{S^\beta}\mathcal{L}\left[\varepsilon T-\left(\alpha+\mu_h\right)R\right]\\
&\mathcal{L}(S_r)=\frac{n_{11}}{S}+\frac{1}{S^\beta}\mathcal{L}\left[\pi_r-\left(\lambda_3+\mu_r\right)S_r\right]\\
&\mathcal{L}(E_r)=\frac{n_{12}}{S}+\frac{1}{S^\beta}\mathcal{L}\left[\lambda_3 S_r-\left(\theta_3+\mu_r\right)E_r\right]\\
&\mathcal{L}(I_r)=\frac{n_{13}}{S}+\frac{1}{S^\beta}\mathcal{L}\left[\theta_3 E_r-\left(\delta_r+\mu_r\right)I_r\right]
\end{aligned}\right\} \tag{8}$$

Decomposing the non-linear term of equation (6) whereby we assume the solution of $S_h(t), E_c(t), E_m(t), Q_c(t), Q_m(t), I_c(t), I_m(t), I_{cm}(t), T(t), R(t), S_r(t), E_r\left(t\right), I_r\left(t\right)$ are in the form of infinite series given by:

$$S_h(t)=\sum_{n=0}^{\infty} S_h(n),\quad E_c(t)=\sum_{n=0}^{\infty} E_c(n),\quad E_m(t)=\sum_{n=0}^{\infty} E_m(n),\quad Q_c(t)=\sum_{n=0}^{\infty} Q_c(n),\quad Q_m(t)=\sum_{n=0}^{\infty} Q_m(n),$$
$$G_Q(t)=\sum_{n=0}^{\infty} G_Q(n),\quad I_c(t)=\sum_{n=0}^{\infty} I_c(n),\quad I_m(t)=\sum_{n=0}^{\infty} I_m(n),\quad I_{cm}(t)=\sum_{n=0}^{\infty} I_{cm}(n),\quad R(t)=\sum_{n=0}^{\infty} R(n),$$
$$S_r(t)=\sum_{n=0}^{\infty} S_r(n),\quad E_r(t)=\sum_{n=0}^{\infty} E_r(n),\quad I_r(t)=\sum_{n=0}^{\infty} I_r(n),$$

(9)

We have three (5) non-linear terms. The non-linear term in equation (6) are decomposed by Adomian polynomial as follows:

$$I_c(t)S_h(t)=\sum_{n=0}^{\infty} A(n),\quad I_{cm}(t)S_h(t)=\sum_{n=0}^{\infty} B(n),\quad I_r(t)S_h(t)=\sum_{n=0}^{\infty} C(n),$$
$$I_m(t)S_h(t)=\sum_{n=0}^{\infty} D(n),\quad I_r(t)S_r(t)=\sum_{n=0}^{\infty} E(n)$$

(10)

Where $A(n), B(n), C(n), D(n), E\left(n\right)$ are Adomian polynomials given by

$$A(n)=\frac{1}{\Gamma(n+1)}\frac{d^n}{d\lambda^n}\left[\sum_{k=0}^{n}\lambda^k I_c(k)\sum_{k=0}^{n}\lambda^k S_h(k)\right]_{\lambda=0}$$

$$B(n)=\frac{1}{\Gamma(n+1)}\frac{d^n}{d\lambda^n}\left[\sum_{k=0}^{n}\lambda^k I_{cm}(k)\sum_{k=0}^{n}\lambda^k S_h(k)\right]_{\lambda=0} \qquad (11)$$

$$C(n)=\frac{1}{\Gamma(n+1)}\frac{d^n}{d\lambda^n}\left[\sum_{k=0}^{n}\lambda^k I_r(k)\sum_{k=0}^{n}\lambda^k S_h(k)\right]_{\lambda=0}$$

$$D(n)=\frac{1}{\Gamma(n+1)}\frac{d^n}{d\lambda^n}\left[\sum_{k=0}^{n}\lambda^k I_m(k)\sum_{k=0}^{n}\lambda^k S_h(k)\right]_{\lambda=0}$$

$$E(n)=\frac{1}{\Gamma(n+1)}\frac{d^n}{d\lambda^n}\left[\sum_{k=0}^{n}\lambda^k I_r(k)\sum_{k=0}^{n}\lambda^k S_r(k)\right]_{\lambda=0}$$

The polynomials are given by

$$A(0) = I_c(0)S_h(0),$$
$$A(1) = I_c(0)S_h(1) + I_c(1)S_h(0),$$
$$A(2) = I_c(0)S_h(2) + I_c(1)S_h(1) + I_c(2)S_h(0).$$

$$B(0) = I_{cm}(0)S_h(0),$$
$$B(1) = I_{cm}(0)S_h(1) + I_{cm}(1)S_h(0),$$
$$B(2) = I_{cm}(0)S_h(2) + I_{cm}(1)S_h(1) + I_{cm}(2)S_h(0).$$

$$C(0) = I_r(0)S_h(0),$$
$$C(1) = I_r(0)S_h(1) + I_r(1)S_h(0),$$
$$C(2) = I_r(0)S_h(2) + I_r(1)S_h(1) + I_r(2)S_h(0).$$

$$D(0) = I_m(0)S_h(0),$$
$$D(1) = I_m(0)S_h(1) + I_m(1)S_h(0), \qquad (12)$$
$$D(2) = I_m(0)S_h(2) + I_m(1)S_h(1) + I_m(2)S_h(0).$$

$$E(0) = I_r(0)S_r(0),$$
$$E(1) = I_r(0)S_r(1) + I_r(1)S_r(0),$$
$$E(2) = I_r(0)S_r(2) + I_r(1)S_r(1) + I_r(2)S_r(0).$$

Substituting equation (9), (10) into equation (8) we obtained:

$$
\left.\begin{aligned}
&\mathcal{L}\left\{\sum_{n=0}^{\infty} S_h(n)\right\}=\frac{n_1}{S}+\frac{1}{S^{\beta}}\mathcal{L}\left[\pi_h+\alpha\sum_{n=0}^{\infty} R(n)+\omega_1 Q_c(n)+\omega_2\sum_{n=0}^{\infty} Q_m(n)-\left(\frac{\beta_c\left(\sum_{n=0}^{\infty} A(n)+\sum_{n=0}^{\infty} B(n)\right)}{N_h}+\frac{\beta_1\sum_{n=0}^{\infty} C(n)+\beta_m\left(\sum_{n=0}^{\infty} D(n)+\sum_{n=0}^{\infty} B(n)\right)}{N_h}\right)-\mu_h\sum_{n=0}^{\infty} S_h(n)\right]\\
&\mathcal{L}\left\{\sum_{n=0}^{\infty} E_c(n)\right\}=\frac{n_2}{S}+\frac{1}{S^{\beta}}\mathcal{L}\left[\frac{\beta_c\left(\sum_{n=0}^{\infty} A(n)+\sum_{n=0}^{\infty} B(n)\right)}{N_h}-(\phi_1+\theta_1+\mu_h)\sum_{n=0}^{\infty} E_c(n)\right]\\
&\mathcal{L}\left\{\sum_{n=0}^{\infty} E_m(n)\right\}=\frac{n_3}{S}+\frac{1}{S^{\beta}}\mathcal{L}\left[\frac{\beta_1\sum_{n=0}^{\infty} C(n)+\beta_m\left(\sum_{n=0}^{\infty} D(n)+\sum_{n=0}^{\infty} B(n)\right)}{N_h}-(\phi_2+\theta_2+\mu_h)\sum_{n=0}^{\infty} E_m(n)\right]\\
&\mathcal{L}\left\{\sum_{n=0}^{\infty} Q_c(n)\right\}=\frac{n_4}{S}+\frac{1}{S^{\beta}}\mathcal{L}\left[\phi_1\sum_{n=0}^{\infty} E_c(n)-(\omega_1+\gamma_4+\delta_1+\mu_h)\sum_{n=0}^{\infty} Q_c(n)\right]\\
&\mathcal{L}\left\{\sum_{n=0}^{\infty} Q_m(n)\right\}=\frac{n_5}{S}+\frac{1}{S^{\beta}}\mathcal{L}\left[\phi_2\sum_{n=0}^{\infty} E_m(n)-(\omega_2+\gamma_5+\delta_2+\mu_h)\sum_{n=0}^{\infty} Q_m(n)\right]\\
&\mathcal{L}\left\{\sum_{n=0}^{\infty} I_c(n)\right\}=\frac{n_6}{S}+\frac{1}{S^{\beta}}\mathcal{L}\left[\theta_1\sum_{n=0}^{\infty} E_c(n)-(\tau_1+\gamma_1+\delta_1+\mu_h)\sum_{n=0}^{\infty} I_c(n)\right]\\
&\mathcal{L}\left\{\sum_{n=0}^{\infty} I_m(n)\right\}=\frac{n_7}{S}+\frac{1}{S^{\beta}}\mathcal{L}\left[\theta_2\sum_{n=0}^{\infty} E_m(n)-(\tau_2+\gamma_3+\delta_2+\mu_h)\sum_{n=0}^{\infty} I_m(n)\right]\\
&\mathcal{L}\left\{\sum_{n=0}^{\infty} I_{cm}(n)\right\}=\frac{n_8}{S}+\frac{1}{S^{\beta}}\mathcal{L}\left[\tau_1\sum_{n=0}^{\infty} I_c(n)+\tau_2\sum_{n=0}^{\infty} I_m(n)-(\gamma_2+\delta_3+\mu_h)\sum_{n=0}^{\infty} I_{cm}(n)\right]\\
&\mathcal{L}\left\{\sum_{n=0}^{\infty} T(n)\right\}=\frac{n_9}{S}+\frac{1}{S^{\beta}}\mathcal{L}\left[\gamma_1\sum_{n=0}^{\infty} I_c(n)+\gamma_2\sum_{n=0}^{\infty} I_{cm}(n)+\gamma_3\sum_{n=0}^{\infty} I_m(n)+\gamma_4\sum_{n=0}^{\infty} Q_c(n)+\gamma_5\sum_{n=0}^{\infty} Q_m(n)-(\varepsilon+\psi\delta_4+\mu_h)\sum_{n=0}^{\infty} T(n)\right]\\
&\mathcal{L}\left\{\sum_{n=0}^{\infty} R(n)\right\}=\frac{n_{10}}{S}+\frac{1}{S^{\beta}}\mathcal{L}\left[\varepsilon\sum_{n=0}^{\infty} T(n)-(\alpha+\mu_h)\sum_{n=0}^{\infty} R(n)\right]\\
&\mathcal{L}\left\{\sum_{n=0}^{\infty} S_r(n)\right\}=\frac{n_{11}}{S}+\frac{1}{S^{\beta}}\mathcal{L}\left[\pi_r-\frac{\beta_r\sum_{n=0}^{\infty} E(n)}{N_r}-\mu_r\sum_{n=0}^{\infty} S_r(n)\right]\\
&\mathcal{L}\left\{\sum_{n=0}^{\infty} E_r(n)\right\}=\frac{n_{12}}{S}+\frac{1}{S^{\beta}}\mathcal{L}\left[\frac{\beta_r\sum_{n=0}^{\infty} E(n)}{N_r}-(\theta_3+\mu_r)\sum_{n=0}^{\infty} E_r(n)\right]\\
&\mathcal{L}\left\{\sum_{n=0}^{\infty} I_r(n)\right\}=\frac{n_{13}}{S}+\frac{1}{S^{\beta}}\mathcal{L}\left[\theta_3\sum_{n=0}^{\infty} E_r(n)-(\delta_r+\mu_r)\sum_{n=0}^{\infty} I_r(n)\right]
\end{aligned}\right\} \tag{13}
$$

Evaluating the Laplace transform of the 2[nd] terms in the RHS of (16), we obtain

$$
\left.\begin{aligned}
&\mathcal{L}\left\{\sum_{n=0}^{\infty} S_h(n)\right\}=\frac{n_1}{S}+\left[\pi_h+\alpha\sum_{n=0}^{\infty} R(n)+\omega_1 Q_c(n)+\omega_2\sum_{n=0}^{\infty} Q_m(n)-\left(\frac{\beta_c\left(\sum_{n=0}^{\infty} A(n)+\sum_{n=0}^{\infty} B(n)\right)}{N_h}+\frac{\beta_1\sum_{n=0}^{\infty} C(n)+\beta_m\left(\sum_{n=0}^{\infty} D(n)+\sum_{n=0}^{\infty} B(n)\right)}{N_h}\right)-\mu_h\sum_{n=0}^{\infty} S_h(n)\right]\frac{1}{S^{\beta+1}}\\
&\mathcal{L}\left\{\sum_{n=0}^{\infty} E_c(n)\right\}=\frac{n_2}{S}+\left[\frac{\beta_c\left(\sum_{n=0}^{\infty} A(n)+\sum_{n=0}^{\infty} B(n)\right)}{N_h}-(\phi_1+\theta_1+\mu_h)\sum_{n=0}^{\infty} E_c(n)\right]\frac{1}{S^{\beta+1}}\\
&\mathcal{L}\left\{\sum_{n=0}^{\infty} E_m(n)\right\}=\frac{n_3}{S}+\left[\frac{\beta_1\sum_{n=0}^{\infty} C(n)+\beta_m\left(\sum_{n=0}^{\infty} D(n)+\sum_{n=0}^{\infty} B(n)\right)}{N_h}-(\phi_2+\theta_2+\mu_h)\sum_{n=0}^{\infty} E_m(n)\right]\frac{1}{S^{\beta+1}}\\
&\mathcal{L}\left\{\sum_{n=0}^{\infty} Q_c(n)\right\}=\frac{n_4}{S}+\mathcal{L}\left[\phi_1\sum_{n=0}^{\infty} E_c(n)-(\omega_1+\gamma_4+\delta_1+\mu_h)\sum_{n=0}^{\infty} Q_c(n)\right]\frac{1}{S^{\beta+1}}\\
&\mathcal{L}\left\{\sum_{n=0}^{\infty} Q_m(n)\right\}=\frac{n_5}{S}+\mathcal{L}\left[\phi_2\sum_{n=0}^{\infty} E_m(n)-(\omega_2+\gamma_5+\delta_2+\mu_h)\sum_{n=0}^{\infty} Q_m(n)\right]\frac{1}{S^{\beta+1}}\\
&\mathcal{L}\left\{\sum_{n=0}^{\infty} I_c(n)\right\}=\frac{n_6}{S}+\mathcal{L}\left[\theta_1\sum_{n=0}^{\infty} E_c(n)-(\tau_1+\gamma_1+\delta_1+\mu_h)\sum_{n=0}^{\infty} I_c(n)\right]\frac{1}{S^{\beta+1}}\\
&\mathcal{L}\left\{\sum_{n=0}^{\infty} I_m(n)\right\}=\frac{n_7}{S}+\mathcal{L}\left[\theta_2\sum_{n=0}^{\infty} E_m(n)-(\tau_2+\gamma_3+\delta_2+\mu_h)\sum_{n=0}^{\infty} I_m(n)\right]\frac{1}{S^{\beta+1}}\\
&\mathcal{L}\left\{\sum_{n=0}^{\infty} I_{cm}(n)\right\}=\frac{n_8}{S}+\mathcal{L}\left[\tau_1\sum_{n=0}^{\infty} I_c(n)+\tau_2\sum_{n=0}^{\infty} I_m(n)-(\gamma_2+\delta_3+\mu_h)\sum_{n=0}^{\infty} I_{cm}(n)\right]\frac{1}{S^{\beta+1}}\\
&\mathcal{L}\left\{\sum_{n=0}^{\infty} T(n)\right\}=\frac{n_9}{S}+\mathcal{L}\left[\gamma_1\sum_{n=0}^{\infty} I_c(n)+\gamma_2\sum_{n=0}^{\infty} I_{cm}(n)+\gamma_3\sum_{n=0}^{\infty} I_m(n)+\gamma_4\sum_{n=0}^{\infty} Q_c(n)+\gamma_5\sum_{n=0}^{\infty} Q_m(n)-(\varepsilon+\psi\delta_4+\mu_h)\sum_{n=0}^{\infty} T(n)\right]\frac{1}{S^{\beta+1}}\\
&\mathcal{L}\left\{\sum_{n=0}^{\infty} R(n)\right\}=\frac{n_{10}}{S}+\mathcal{L}\left[\varepsilon\sum_{n=0}^{\infty} T(n)-(\alpha+\mu_h)\sum_{n=0}^{\infty} R(n)\right]\frac{1}{S^{\beta+1}}\\
&\mathcal{L}\left\{\sum_{n=0}^{\infty} S_r(n)\right\}=\frac{n_{11}}{S}+\mathcal{L}\left[\pi_r-\frac{\beta_r\sum_{n=0}^{\infty} E(n)}{N_r}-\mu_r\sum_{n=0}^{\infty} S_r(n)\right]\frac{1}{S^{\beta+1}}\\
&\mathcal{L}\left\{\sum_{n=0}^{\infty} E_r(n)\right\}=\frac{n_{12}}{S}+\mathcal{L}\left[\frac{\beta_r\sum_{n=0}^{\infty} E(n)}{N_r}-(\theta_3+\mu_r)\sum_{n=0}^{\infty} E_r(n)\right]\frac{1}{S^{\beta+1}}\\
&\mathcal{L}\left\{\sum_{n=0}^{\infty} I_r(n)\right\}=\frac{n_{13}}{S}+\mathcal{L}\left[\theta_3\sum_{n=0}^{\infty} E_r(n)-(\delta_r+\mu_r)\sum_{n=0}^{\infty} I_r(n)\right]\frac{1}{S^{\beta+1}}
\end{aligned}\right\} \tag{14}
$$

Taking the inverse Laplace transform of both sides of (14)

$$
\left.
\begin{aligned}
&\sum_{n=0}^{\infty} S_h(n)=n_1+\left[\pi_h+\alpha\sum_{n=0}^{\infty} R(n)+\omega_1 Q_c(n)+\omega_2\sum_{n=0}^{\infty} Q_m(n)-\left(\frac{\beta_c\left(\sum_{n=0}^{\infty} A(n)+\sum_{n=0}^{\infty} B(n)\right)}{N_h}+\frac{\beta_1\sum_{n=0}^{\infty} C(n)+\beta_m\left(\sum_{n=0}^{\infty} D(n)+\sum_{n=0}^{\infty} B(n)\right)}{N_h}\right)-\mu_h\sum_{n=0}^{\infty} S_h(n)\right]\frac{t^{\beta}}{\Gamma(\beta+1)}\\
&\sum_{n=0}^{\infty} E_c(n)=n_2+\left[\frac{\beta_c\left(\sum_{n=0}^{\infty} A(n)+\sum_{n=0}^{\infty} B(n)\right)}{N_h}-(\phi_1+\theta_1+\mu_h)\sum_{n=0}^{\infty} E_c(n)\right]\frac{t^{\beta}}{\Gamma(\beta+1)}\\
&\sum_{n=0}^{\infty} E_m(n)=n_3+\left[\frac{\beta_1\sum_{n=0}^{\infty} C(n)+\beta_m\left(\sum_{n=0}^{\infty} D(n)+\sum_{n=0}^{\infty} B(n)\right)}{N_h}-(\phi_2+\theta_2+\mu_h)\sum_{n=0}^{\infty} E_m(n)\right]\frac{1}{S^{\beta+1}}\\
&\sum_{n=0}^{\infty} Q_c(n)=n_4+\mathcal{L}\left[\phi_1\sum_{n=0}^{\infty} E_c(n)-(\omega_1+\gamma_4+\delta_1+\mu_h)\sum_{n=0}^{\infty} Q_c(n)\right]\frac{1}{S^{\beta+1}}\\
&\sum_{n=0}^{\infty} Q_m(n)=n_5+\mathcal{L}\left[\phi_2\sum_{n=0}^{\infty} E_m(n)-(\omega_2+\gamma_5+\delta_2+\mu_h)\sum_{n=0}^{\infty} Q_m(n)\right]\frac{1}{S^{\beta+1}}\\
&\sum_{n=0}^{\infty} I_c(n)=n_6+\mathcal{L}\left[\theta_1\sum_{n=0}^{\infty} E_c(n)-(\tau_1+\gamma_1+\delta_1+\mu_h)\sum_{n=0}^{\infty} I_c(n)\right]\frac{1}{S^{\beta+1}}\\
&\sum_{n=0}^{\infty} I_m(n)=n_7+\mathcal{L}\left[\theta_2\sum_{n=0}^{\infty} E_m(n)-(\tau_2+\gamma_3+\delta_2+\mu_h)\sum_{n=0}^{\infty} I_m(n)\right]\frac{1}{S^{\beta+1}}\\
&\sum_{n=0}^{\infty} I_{cm}(n)=n_8+\mathcal{L}\left[\tau_1\sum_{n=0}^{\infty} I_c(n)+\tau_2\sum_{n=0}^{\infty} I_m(n)-(\gamma_2+\delta_3+\mu_h)\sum_{n=0}^{\infty} I_{cm}(n)\right]\frac{1}{S^{\beta+1}}\\
&\sum_{n=0}^{\infty} T(n)=n_9+\mathcal{L}\left[\gamma_1\sum_{n=0}^{\infty} I_c(n)+\gamma_2\sum_{n=0}^{\infty} I_{cm}(n)+\gamma_3\sum_{n=0}^{\infty} I_m(n)+\gamma_4\sum_{n=0}^{\infty} Q_c(n)+\gamma_5\sum_{n=0}^{\infty} Q_m(n)-(\varepsilon+\psi\delta_4+\mu_h)\sum_{n=0}^{\infty} T(n)\right]\frac{1}{S^{\beta+1}}\\
&\sum_{n=0}^{\infty} R(n)=n_{10}+\mathcal{L}\left[\varepsilon\sum_{n=0}^{\infty} T(n)-(\alpha+\mu_h)\sum_{n=0}^{\infty} R(n)\right]\frac{1}{S^{\beta+1}}\\
&\sum_{n=0}^{\infty} S_r(n)=n_{11}+\mathcal{L}\left[\pi_r-\frac{\beta_r\sum_{n=0}^{\infty} E(n)}{N_r}-\mu_r\sum_{n=0}^{\infty} S_r(n)\right]\frac{1}{S^{\beta+1}}\\
&\sum_{n=0}^{\infty} E_r(n)=n_{12}+\mathcal{L}\left[\frac{\beta_r\sum_{n=0}^{\infty} E(n)}{N_r}-(\theta_3+\mu_r)\sum_{n=0}^{\infty} E_r(n)\right]\frac{1}{S^{\beta+1}}\\
&\sum_{n=0}^{\infty} I_r(n)=n_{13}+\mathcal{L}\left[\theta_3\sum_{n=0}^{\infty} E_r(n)-(\delta_r+\mu_r)\sum_{n=0}^{\infty} I_r(n)\right]\frac{1}{S^{\beta+1}}
\end{aligned}
\right\}
\tag{15}
$$

When $n=0$ we obtain,

$$S_h(0)=n_1,\ \ E_c(0)=n_2,\ E_m(0)=n_3,\ Q_c(0)=n_4,\ Q_m(0)=n_5,\ I_c(0)=n_6,\ I_m(0)=n_7,$$

$$I_{cm}(0)=n_8,\ T(0)=n_9,\ R(0)=n_{10},\ S_r(0)=n_{11}, E_r(0)=n_{12}, I_r(0)=n_{13}$$

(16)

When $n=1$, we obtain,

$$\left.\begin{aligned}
&S_h(1)=\left[\pi_h+\alpha R(0)+\omega_1 Q_c(0)+\omega_2 Q_m(0)-\left(\frac{\beta_c\left(A(0)+B(0)\right)}{N_h}+\frac{\beta_1 C(0)+\beta_m\left(D(0)+B(0)\right)}{N_h}\right)-\mu_h S_h(0)\right]\frac{t^\beta}{\Gamma(\beta+1)}\\
&E_c(1)=\left[\frac{\beta_c\left(A(0)+B(0)\right)}{N_h}-\left(\phi_1+\theta_1+\mu_h\right)E_c(0)\right]\frac{t^\beta}{\Gamma(\beta+1)}\\
&E_m(1)=\left[\frac{\beta_1 C(0)+\beta_m\left(D(0)+B(0)\right)}{N_h}-\left(\phi_2+\theta_2+\mu_h\right)E_m(0)\right]\frac{1}{S^{\beta+1}}\\
&Q_c(1)=\left[\phi_1 E_c(0)-\left(\omega_1+\gamma_4+\delta_1+\mu_h\right)Q_c(0)\right]\frac{1}{S^{\beta+1}}\\
&Q_m(1)=\left[\phi_2 E_m(0)-\left(\omega_2+\gamma_5+\delta_2+\mu_h\right)Q_m(0)\right]\frac{1}{S^{\beta+1}}\\
&I_c(1)=\left[\theta_1 E_c(0)-\left(\tau_1+\gamma_1+\delta_1+\mu_h\right)I_c(0)\right]\frac{1}{S^{\beta+1}}\\
&I_m(1)=\left[\theta_2 E_m(0)-\left(\tau_2+\gamma_3+\delta_2+\mu_h\right)I_m(0)\right]\frac{1}{S^{\beta+1}}\\
&I_{cm}(1)=\left[\tau_1 I_c(0)+\tau_2 I_m(0)-\left(\gamma_2+\delta_3+\mu_h\right)I_{cm}(0)\right]\frac{1}{S^{\beta+1}}\\
&T(1)=\left[\gamma_1 I_c(0)+\gamma_2 I_{cm}(0)+\gamma_3 I_m(0)+\gamma_4 Q_c(0)+\gamma_5 Q_m(0)-\left(\varepsilon+\psi\delta_4+\mu_h\right)T(0)\right]\frac{1}{S^{\beta+1}}\\
&R(1)=\left[\varepsilon T(0)-\left(\alpha+\mu_h\right)R(0)\right]\frac{1}{S^{\beta+1}}\\
&S_r(1)=\left[\pi_r-\frac{\beta_r E(0)}{N_r}-\mu_r S_r(0)\right]\frac{1}{S^{\beta+1}}\\
&E_r(1)=\left[\frac{\beta_r E(0)}{N_r}-\left(\theta_3+\mu_r\right)E_r(0)\right]\frac{1}{S^{\beta+1}}\\
&I_r(1)=\left[\theta_3 E_r(0)-\left(\delta_r+\mu_r\right)I_r(0)\right]\frac{1}{S^{\beta+1}}
\end{aligned}\right\}\qquad (17)$$

When $n=2$, we obtain,

$$\left.\begin{array}{l}
S_h(2)=\left[\pi_h+\alpha R(1)+\omega_1 Q_c(1)+\omega_2 Q_m(1)-\left(\frac{\beta_c\left(A(1)+B(1)\right)}{N_h}+\frac{\beta_1 C(1)+\beta_m\left(D(1)+B(1)\right)}{N_h}\right)-\mu_h S_h(1)\right]\frac{t^{\beta}}{\Gamma(\beta+1)} \\
E_c(2)=\left[\frac{\beta_c\left(A(1)+B(1)\right)}{N_h}-\left(\phi_1+\theta_1+\mu_h\right)E_c(1)\right]\frac{t^{\beta}}{\Gamma(\beta+1)} \\
E_m(2)=\left[\frac{\beta_1 C(1)+\beta_m\left(D(1)+B(1)\right)}{N_h}-\left(\phi_2+\theta_2+\mu_h\right)E_m(1)\right]\frac{1}{S^{\beta+1}} \\
Q_c(2)=\left[\phi_1 E_c(1)-\left(\omega_1+\gamma_4+\delta_1+\mu_h\right)Q_c(1)\right]\frac{1}{S^{\beta+1}} \\
Q_m(2)=\left[\phi_2 E_m(1)-\left(\omega_2+\gamma_5+\delta_2+\mu_h\right)Q_m(1)\right]\frac{1}{S^{\beta+1}} \\
I_c(2)=\left[\theta_1 E_c(1)-\left(\tau_1+\gamma_1+\delta_1+\mu_h\right)I_c(1)\right]\frac{1}{S^{\beta+1}} \\
I_m(2)=\left[\theta_2 E_m(1)-\left(\tau_2+\gamma_3+\delta_2+\mu_h\right)I_m(1)\right]\frac{1}{S^{\beta+1}} \\
I_{cm}(2)=\left[\tau_1 I_c(1)+\tau_2 I_m(1)-\left(\gamma_2+\delta_3+\mu_h\right)I_{cm}(1)\right]\frac{1}{S^{\beta+1}} \\
T(2)=\left[\gamma_1 I_c(1)+\gamma_2 I_{cm}(1)+\gamma_3 I_m(1)+\gamma_4 Q_c(1)+\gamma_5 Q_m(1)-\left(\varepsilon+\psi\delta_4+\mu_h\right)T(1)\right]\frac{1}{S^{\beta+1}} \\
R(2)=\left[\varepsilon T(1)-\left(\alpha+\mu_h\right)R(1)\right]\frac{1}{S^{\beta+1}} \\
S_r(2)=\left[\pi_r-\frac{\beta_r E(1)}{N_r}-\mu_r S_r(1)\right]\frac{1}{S^{\beta+1}} \\
E_r(2)=\left[\frac{\beta_r E(1)}{N_r}-\left(\theta_3+\mu_r\right)E_r(1)\right]\frac{1}{S^{\beta+1}} \\
I_r(2)=\left[\theta_3 E_r(1)-\left(\delta_r+\mu_r\right)I_r(1)\right]\frac{1}{S^{\beta+1}}
\end{array}\right\} \tag{18}$$

$$\vdots \quad = \quad \vdots$$

When $n=n+1$, we obtain,

$$\left.\begin{aligned}
&S_h(n+1)=\left[\pi_h+\alpha R(n)+\omega_1 Q_c(n)+\omega_2 Q_m(n)-\left(\frac{\beta_c\left(A(n)+B(n)\right)}{N_h}+\frac{\beta_1 C(n)+\beta_m\left(D(n)+B(n)\right)}{N_h}\right)-\mu_h S_h(n)\right]\frac{t^{\beta}}{\Gamma(\beta+1)}\\
&E_c(n+1)=\left[\frac{\beta_c\left(A(n)+B(n)\right)}{N_h}-(\phi_1+\theta_1+\mu_h)E_c(n)\right]\frac{t^{\beta}}{\Gamma(\beta+1)}\\
&E_m(n+1)=\left[\frac{\beta_1 C(n)+\beta_m\left(D(n)+B(n)\right)}{N_h}-(\phi_2+\theta_2+\mu_h)E_m(n)\right]\frac{1}{S^{\beta+1}}\\
&Q_c(n+1)=\left[\phi_1 E_c(n)-(\omega_1+\gamma_4+\delta_1+\mu_h)Q_c(n)\right]\frac{1}{S^{\beta+1}}\\
&Q_m(n+1)=\left[\phi_2 E_m(n)-(\omega_2+\gamma_5+\delta_2+\mu_h)Q_m(n)\right]\frac{1}{S^{\beta+1}}\\
&I_c(n+1)=\left[\theta_1 E_c(n)-(\tau_1+\gamma_1+\delta_1+\mu_h)I_c(n)\right]\frac{1}{S^{\beta+1}}\\
&I_m(n+1)=\left[\theta_2 E_m(n)-(\tau_2+\gamma_3+\delta_2+\mu_h)I_m(n)\right]\frac{1}{S^{\beta+1}}\\
&I_{cm}(n+1)=\left[\tau_1 I_c(n)+\tau_2 I_m(n)-(\gamma_2+\delta_3+\mu_h)I_{cm}(n)\right]\frac{1}{S^{\beta+1}}\\
&T(n+1)=\left[\gamma_1 I_c(n)+\gamma_2 I_{cm}(n)+\gamma_3 I_m(n)+\gamma_4 Q_c(n)+\gamma_5 Q_m(n)-(\varepsilon+\psi\delta_4+\mu_h)T(n)\right]\frac{1}{S^{\beta+1}}\\
&R(n+1)=\left[\varepsilon T(n)-(\alpha+\mu_h)R(n)\right]\frac{1}{S^{\beta+1}}\\
&S_r(n+1)=\left[\pi_r-\frac{\beta_r E(n)}{N_r}-\mu_r S_r(n)\right]\frac{1}{S^{\beta+1}}\\
&E_r(n+1)=\left[\frac{\beta_r E(n)}{N_r}-(\theta_3+\mu_r)E_r(n)\right]\frac{1}{S^{\beta+1}}\\
&I_r(n+1)=\left[\theta_3 E_r(n)-(\delta_r+\mu_r)I_r(n)\right]\frac{1}{S^{\beta+1}}
\end{aligned}\right\} \tag{19}$$

The series solution of each compartment can be expressed as:

$$S_h(t)=S_h(0)+S_h(1)+S_h(2)+\ldots$$

$$E_c(t)=E_c(0)+E_c(1)+E_c(2)+\ldots$$

$$E_m(t) = E_m(0) + E_m(1) + E_m(2) + \ldots$$

$$Q_c(t) = Q_c(0) + Q_c(1) + Q_c(2) + \ldots$$

$$Q_m(t) = Q_m(0) + Q_m(1) + Q_m(2) + \ldots \quad (20)$$

$$I_c(t) = I_c(0) + I_c(1) + I_c(2) + \ldots$$

$$I_m(t) = I_m(0) + I_m(1) + I_m(2) + \ldots$$

$$I_{cm}(t) = I_{cm}(0) + I_{cm}(1) + I_{cm}(2) + \ldots$$

$$T(t) = T(0) + T(1) + T(2) + \ldots$$

$$R(t) = R(0) + R(1) + R(2) + \ldots$$

$$S_r(t) = S_r(0) + S_r(1) + S_r(2) + \ldots$$

$$E_r(t) = E_r(0) + E_r(1) + E_r(2) + \ldots$$

$$I_r(t) = I_r(0) + I_r(1) + I_r(2) + \ldots$$

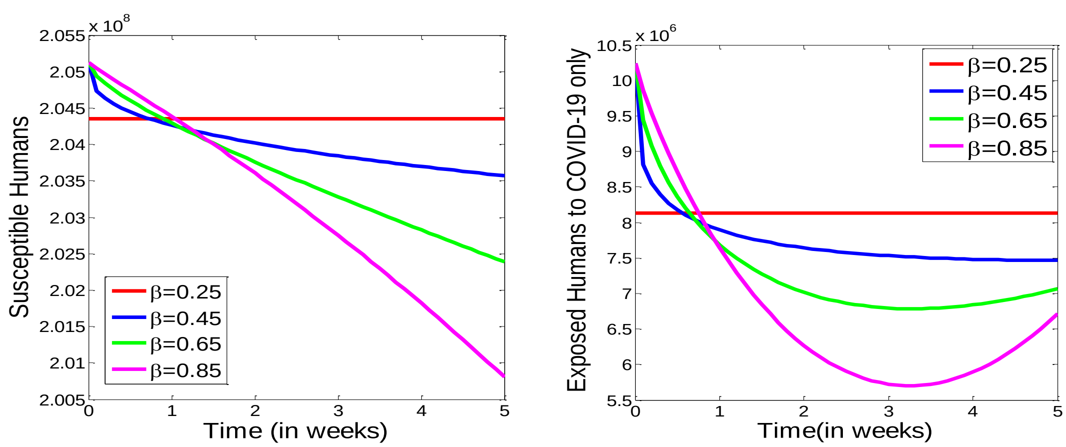
Numerical Solution of Laplace Adomian Decomposition Method (LADM)

In this section, we will see the numerical solution of the model. Using the initial conditions, the Laplace Adomian Decomposition Method (LADM) gives us an approximate solution in in terms of an infinite series presented as:

$$\left.\begin{aligned}
&S_h(t)=205,125,000-630,233.89\frac{t^{\beta}}{\Gamma(\beta+1)}-141,373.21\frac{t^{2\beta}}{\Gamma(2\beta+1)}+\ldots\\
&E_c(t)=10,243,000-3,024,055.73\frac{t^{\beta}}{\Gamma(\beta+1)}+912,356.18\frac{t^{2\beta}}{\Gamma(2\beta+1)}+\ldots\\
&E_m(t)=10,243,000-2,224,088.07\frac{t^{\beta}}{\Gamma(\beta+1)}+632,059.11\frac{t^{2\beta}}{\Gamma(2\beta+1)}+\ldots\\
&Q_c(t)=70,000+953,695.90\frac{t^{\beta}}{\Gamma(\beta+1)}-1,264,331.90\frac{t^{2\beta}}{\Gamma(2\beta+1)}+\ldots\\
&Q_m(t)=10,000+581,444.00\frac{t^{\beta}}{\Gamma(\beta+1)}-326,506.31\frac{t^{2\beta}}{\Gamma(2\beta+1)}+\ldots\\
&I_c(t)=100,000+190,8374.69\frac{t^{\beta}}{\Gamma(\beta+1)}-1,753,921.00\frac{t^{2\beta}}{\Gamma(2\beta+1)}+\ldots\\
&I_m(t)=80,000+1,587,295.00\frac{t^{\beta}}{\Gamma(\beta+1)}-927,355.53\frac{t^{2\beta}}{\Gamma(2\beta+1)}+\ldots\\
&I_{cm}(t)=10,000+36,586.04\frac{t^{\beta}}{\Gamma(\beta+1)}+766,008.36\frac{t^{2\beta}}{\Gamma(2\beta+1)}+\ldots\\
&T(t)=100,000+23,829.70\frac{t^{\beta}}{\Gamma(\beta+1)}+494,396.16\frac{t^{2\beta}}{\Gamma(2\beta+1)}+\ldots\\
&R(t)=19,000+7,690.43\frac{t^{\beta}}{\Gamma(\beta+1)}+1,797.72\frac{t^{2\beta}}{\Gamma(2\beta+1)}+\ldots\\
&S_r(t)=5,000-210.50\frac{t^{\beta}}{\Gamma(\beta+1)}+69.73\frac{t^{2\beta}}{\Gamma(2\beta+1)}+\ldots\\
&E_r(t)=2,500-4.04\frac{t^{\beta}}{\Gamma(\beta+1)}-68.78\frac{t^{2\beta}}{\Gamma(2\beta+1)}+\ldots\\
&I_r(t)=1,000-302.25\frac{t^{\beta}}{\Gamma(\beta+1)}+151.40\frac{t^{2\beta}}{\Gamma(2\beta+1)}+\ldots
\end{aligned}\right\} \qquad (21)$$

For $\beta=1$, the series solution of our model becomes,

$$\left.\begin{array}{l}
S_h(t) = 205{,}125{,}000 - 630{,}233.89t - 70{,}686.61t^2 + \ldots \\
E_c(t) = 10{,}243{,}000 - 3{,}024{,}055.73t + 456{,}178.09t^2 + \ldots \\
E_m(t) = 10{,}243{,}000 - 2224088.07t + 316{,}029.56t^2 + \ldots \\
Q_c(t) = 70{,}000 + 953695.90t - 632{,}165.95t^2 + \ldots \\
Q_m(t) = 10{,}000 + 581444.00t - 163{,}253.155t^2 + \ldots \\
I_c(t) = 100{,}000 + 1908374.69t - 876{,}960.5t^2 + \ldots \\
I_m(t) = 80{,}000 + 1587295.00t - 463{,}677.77t^2 + \ldots \\
I_{cm}(t) = 10{,}000 + 36586.04t + 383{,}004.18t^2 + \ldots \\
T(t) = 100{,}000 + 23829.70t + 247{,}198.08t^2 + \ldots \\
R(t) = 19{,}000 + 7690.43t + 898.86t^2 + \ldots \\
S_r(t) = 5{,}000 - 210.50t + 30.365t^2 + \ldots \\
E_r(t) = 2{,}500 - 4.04t - 34.39t^2 + \ldots \\
I_r(t) = 1{,}000 - 302.25t + 75.7t^2 + \ldots
\end{array}\right\} \qquad (22)$$

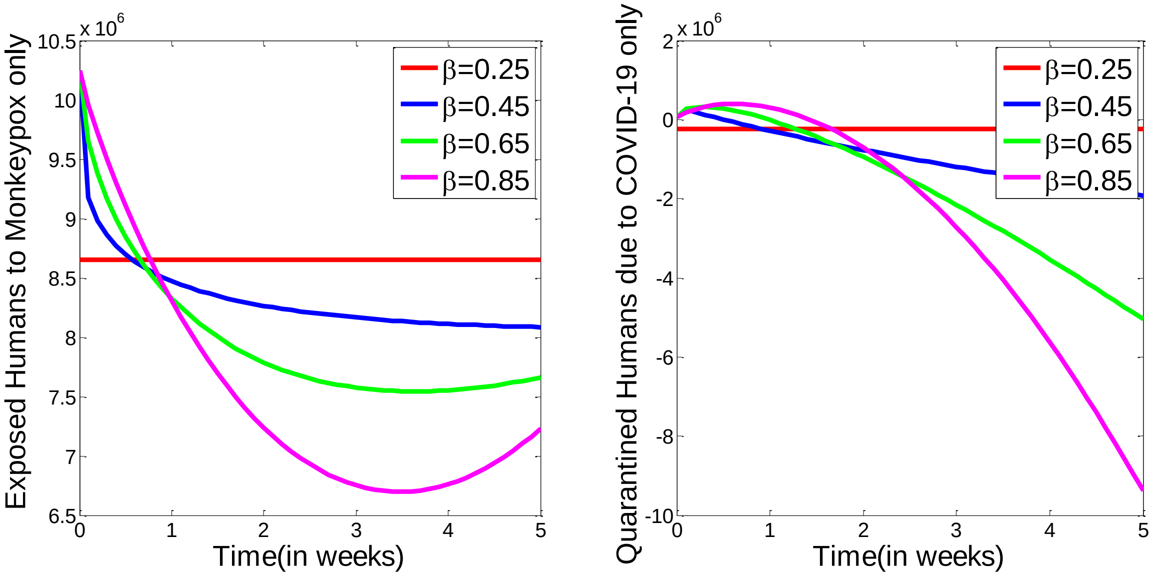


a. Effect of varying β on susceptible Human population

b. Effect of varying β on Exposed COVID-19 humans

Figure 2

From the graph in **figure 2a**, we observed a decrease in the population of the susceptible humans due to their overall progression into the exposed class as a result of their contact with the infected classes of humans. **Figure 2b and 3a** show an initial decrease in the population of the exposed humans to COVID-19 and Monkeypox due to their progression into their respective quarantined classes. This decline is also due to high infectiousness of these diseases resulting into their progression into their respective infected classes and co-infection class.

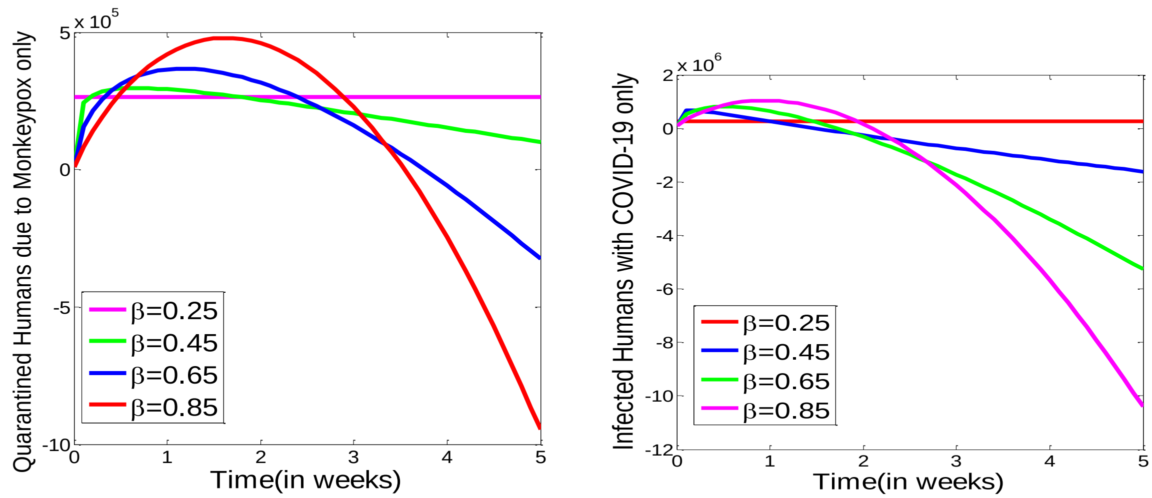


a. Effect of varying β on Exposed Monkeypox population

b. Effect of varying β on Quarantined COVID-19 humans

Figure 3

The graph in **figure 3b and 4a** show a decrease in the population of the quarantined classes due their progression into the infected classes and their discharge into the susceptible classes of humans as a result of no manifestation of clinical symptoms of any of the disease(s).

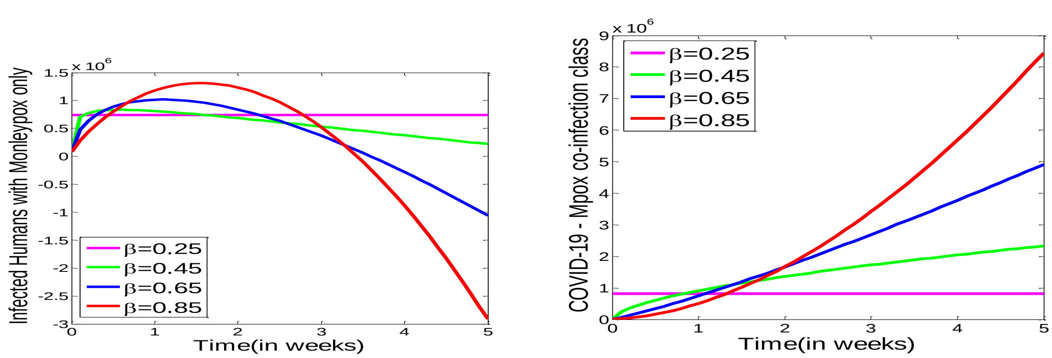


a. Effect of varying β on Quarantined Monkeypox Humans

b. Effect of varying β on Infected COVID-19 humans

Figure 4

Figure 4b and **5a** show an initial increase in the population of the infected classes due to the influx from both the quarantined and exposed classes of these diseases. But at some point, we observed a decrease in the population of the infected classes due to their progression into the co-infection class and also their progression into the treatment classes for a better health care attention.

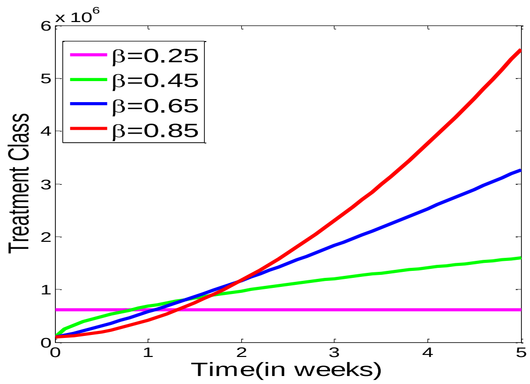


a. Effect of varying β on Infected Monkeypox Humans

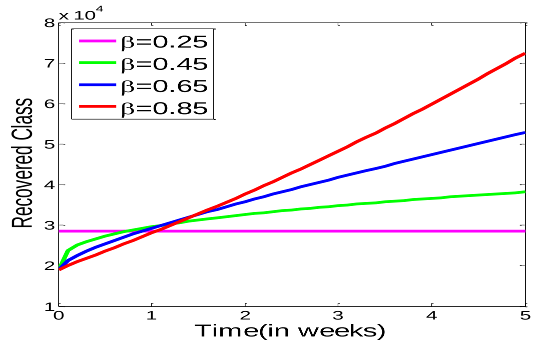
b. Effect of varying β on COVID-19 Mpox co-infection class

Figure 5

Figure 5b shows an increase in the population of the co-infection class of COVID-19 and Monkeypox due to the influx from the two singly infected classes of COVID-19 and Monkeypox.



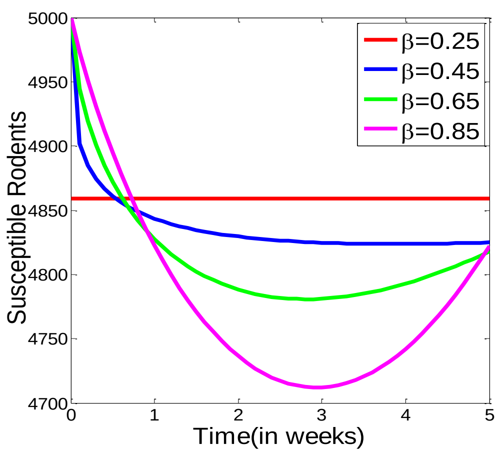
a. Effect of varying β on Treatment Population



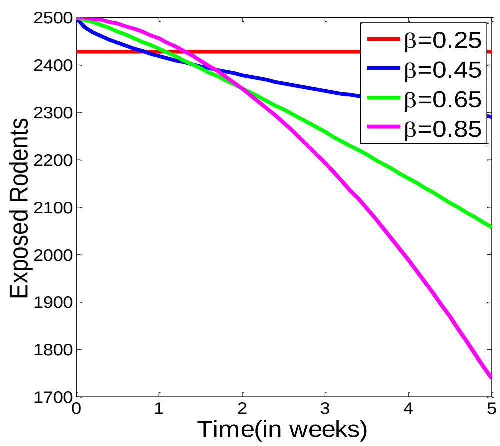
b. Effect of varying β on Recovered Population

Figure 6

Figure 6a and **6b** show an increase in the treatment and recovered classes due to the effective medical attention given to infected individuals which ultimately reduced the burden of COVID-19 and Monkeypox in the human population.



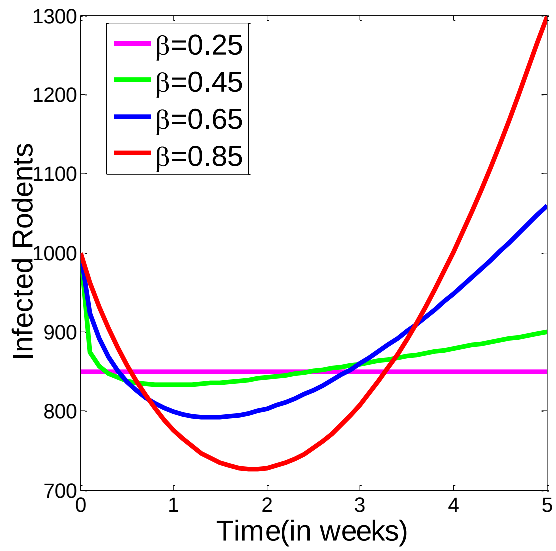
a. Effect of varying β on susceptible Rodents population



b. Effect of varying β on Exposed Rodents Population

Figure 7

Figure 7a shows a decrease in the population of the susceptible rodents due to exposure to rodents infected with Monkeypox. From **figure 7b,** we observed a decrease in the population of the exposed rodents due to the high infectiousness of Monkeypox within the rodent population.



Effect of varying β on Infected Rodents

Figure 8

Figure 8 shows an increase in the population of the infected rodents with Monkeypox due to the high prevalence and burden of the disease in the rodents population.

2.6 Data Fitting for COVID-19 and Monkeypox Models

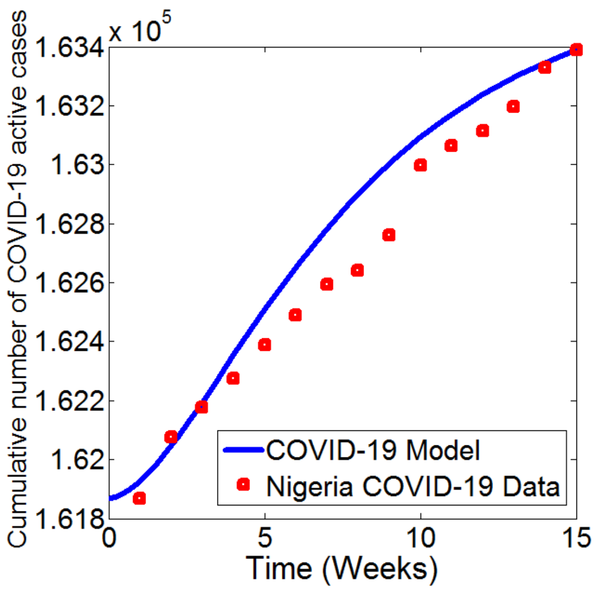
In this section, we describe the method utilized to compute information concerning vital elements in our framework (1). Utilizing the fmincon algorithm from MATLAB's optimization toolkit, we carried out data fitting for our COVID-19 and Monkeypox sub-models. COVID-19 outbreak data was obtained from Nigeria, a nation affected by the COVID-19 crisis, covering the period from July 9, 2021, to August 7, 2021. The table illustrates the documented active cases of COVID-19, while the Monkeypox outbreak data spans from October 2, 2022, to April 23, 2023.

DATE	**Jul. 9**	**Jul. 10**	**Jul. 11**	**Jul.12**	**Jul. 13**
CASES	11,713	11,515	11,421	10,357	10,363
DATE	**Jul. 14**	**Jul. 15**	**Jul. 16**	**Jul. 17**	**Jul. 18**
CASES	10,243	10,237	10,126	9174	9231
DATE	**Jul. 19**	**Jul. 20**	**Jul. 21**	**Jul. 22**	**Jul. 23**
CASES	9,170	9,202	9,139	9,227	7,700
DATE	**Jul. 24**	**Jul. 25**	**Jul. 26**	**Jul. 27**	**Jul. 28**
CASES	7,594	7,518	7,520	7,626	

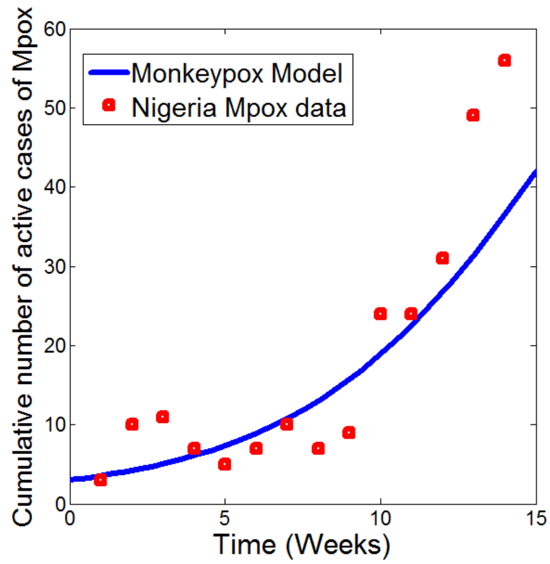
Table 3: COVID-19 Data from Jul. 9 – Jul. 28, 2021

DATE	**CASES**
Oct. 2, 2022	56
Oct. 16, 2022	49
Oct. 30, 2022	31
Dec. 11, 2022	24
Dec. 18, 2022	24
Dec. 25, 2022	9
Jan. 8, 2023	7
Feb. 5, 2023	10
Feb. 19, 2023	7
Mar. 12, 2023	5
Mar. 26, 2023	7
Apr. 9, 2023	11
Apr. 16, 2023	10
Apr. 23, 2023	3

Table 4: Monkeypox data from Oct. 2, 2022 – April 23, 2023



a. COVID-19 Data fitting



b. Monkeypox Data fitting

Figure 9

Parameters	**Value**	**Source**
π_h	0.029	[13]
π_r	0.2	[14]
β_c	0.0109	Fitted
β_m	0.1	Fitted
β_1	0.00025	[15]
β_r	0.3412	Fitted
ω_1	0.5999	Fitted
ω_2	0.04	[14]
ϕ_1	0.1	Fitted
ϕ_2	0.0571	Assumed
θ_1	$\frac{1}{5.2}$	[16]
θ_2	0.1578	Fitted
θ_3	0.0799	Assumed
γ_1	0.2556	Fitted

γ_2	0.088366	Assumed
γ_3	0.01	Fitted
γ_4	0.25	Assumed
γ_5	0.2	[17]
τ_1	0.2	Assumed
τ_2	0.25	Assumed
ε	0.079	Assumed
α	0.008	Fitted
μ_h	0.00303	[14]
μ_r	0.002	[14]
δ_1	0.1557	Fitted
δ_2	0.1001	Fitted
δ_3	0.25	[20]
δ_4	0.1001	Fitted
ψ	0.4	Assumed
δ_r	0.5	[18]

Table 5: Parameters Table of Values

Convergence Analysis for the Laplace-Adomian Decomposition Method (LADM).

The solution of (1) is expressed in the forms of infinite series which converged uniformly to its exact solution. To verify the convergence of the series (21), we employ the method used in [19]. For sufficient conditions of convergence of the LADM, we present the following theorem:

Theorem 1

Let X be a Banach space and $T: X \rightarrow X$ be a constructive nonlinear operator such that for $(x),(x)^{'} \in X$, $\left\|T(x)-T(x^{'})\right\|, 0<k<1$. Then, T has a unique point x such that $Tx = x$, where $x=(S_h, E_c, E_m, Q_c, Q_m, I_c, I_m, I_{cm}, T, R, S_r, E_r, I_r)$. The series given () can be written by applying the Adominan decomposition method as follows:

$$x_n = Tx_{n-1}, x_{n-1},$$

$$= \sum_{i=1}^{n-1} x_i,\ n = 1,2,3,...$$

And we assume that $x_0 \in B_r(x)$, *where* $B_r(x) = \left\{x \in X : \left\|x^{'} - x\right\| < r\right\}$; *then, we have as follows:*

(i) $x_n \in B_r(x)$

(ii) $\lim_{n\to\infty} x_n = x$

Proof

For condition (i), invoking mathematical induction,

For n=1, we have as follows:

$$\|x_0 - x\| = \|T(x_0) - T(x)\| \le \|x_0 - x\|.$$

If this is true for m-1, then

$$\|x_0 - x\| \le k^{m-1}\|x_0 - x\|.$$

This gives the following:

$$\|x_m - x\| = \|T(x_{m-1}) - T(x)\| \le k\|x_{m-1} - x\| \le k^n\|x_0 - x\|.$$

Therefore,

$$\|x_m - x\| \le k^n\|x_0 - x\| \le k^n r < r.$$

This directly implies that $x_n \in B_r(x)$.

Also, for (ii), we have that since $\|x_m - x\| \le k^n\|x_0 - x\|$ and $\lim_{n\to\infty} k^n = 0$, we can write $\lim_{n\to\infty} x_n = x$.

CONCLUSION

In this work, we formulated a fractional order deterministic compartmental model in a bid to study the transmission dynamics on the co-infection of COVID-19 and Monkeypox within the human population. We adopted the well-known Laplace-Adomian Decomposition method in solving and analyzing the formulated model. From our analysis using the aforementioned technique, we obtained series solutions of the co-infection model which were also shown to converge to an exact value. We proceed further to carry out a data fitting analysis so as to obtain the estimates for some key parameters used in the model. It was observed that increasing treatment capacities pose as a pivotal approach in reducing the disease burdens of COVID-19 and Monkeypox and the case of their co-infections within the human populace.

Data Availability

All the data used in the course of this research work has been adequately cited.

Conflicts of Interest

The authors declare that they have no conflict of interest.

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