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Bivariate Time Series Analysis of Nigeria Gross Domestics Product and Communication Sector

Ekpe, Anyanime Udo; Usoro, Anthony Effiong; Ekong Akaninyene Okon.
Department of Statistics, Akwa Ibom State University, Mkpat Enin, Akwa Ibom State, Nigeria

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ABSTRACT: *The need to establish a relation between Gross Domestic Product and Communication sector was the major focus of this research. This research work investigated the contribution of Communication sector to the Gross Domestic product of Nigeria Economy. With the aid of ACF and PACF, ARIMA (1 1 1) was suggested for both variables. Alternative multivariate time series models used for the analysis were ARIMAV, MARDL and MARDL-MA models. The research has established interaction and interdependence between the two macroeconomic variables, and has also revealed that each of the variable has contributed significantly to each other at first time lag. The error variances of the bivariate time series model were derived for GDP and Communication sector. When comparing the three models for the two economic variables, ARIMAV model for Gross Domestic Product has the least error variance of* 0.2183 *making it the best model, while MARDL model for communication sector produced the least error variance of 0.0723, thereby indicating that MARDL model outperformed ARIMAV and MARDL-MA models for communication sector. Hence, this research has brought to focus the fact that performance of a model over another is predicated upon the nature of the economic data. That means there is no fixed multivariate time series model for a given macroeconomic data due to the dynamic nature of the time series.*

KEYWORDS: ARIMAV, MARDL, MARDL-MA, Autocorrelation, Partial autocorrelation

INTRODUCTION

Gross Domestic Product is the market value of all official recognized final goods and services produced within the country in a given period of time. The GDP of a nation is as important as it helps in to show the level of economic development. It is an important indicator to measure a countries' wealth and economy strength. GDP is part of the national income and product account which enable policy makers to determine whether the economy is contracting or expanding and if either a recession or inflation beckons. It is used by economist to determine the level of development of a country. Gross domestic product comprises of Consumption (C), Investment (I), Government (G) Purchase of Goods and Services and net Exports (X) produced within the country during that period. No country or nation can survive without strong Macro-economic indicators such as capital stock, high level of man power, investment, export etc. All these play a significant role in improving and developing the standard and economic system of the country. However, any country with weak macroeconomic indicators may not be able to cope with the current global economic meltdown. There is no dispute as to the fact that, growth and stability of a nation's economy lie on its Gross Domestic Product (GDP) as a measure of economic wealth and its relationship with key economic sectors. Nigerian Economy is said to be a mixed and emerging market with numerous expanding economic sectors such as Oil & Gas, Manufacturing, Finance, Service, Communications, Technology and Entertainment sectors, etc., both private and public (Frankfurt, 2020*)*. It is ranked as the 27th-largest economy in the world in terms of GDP, and the 24th-largest in terms of purchasing power parity (World Bank, 2020). As fondly called the giant of Africa, Nigeria has the largest economy in Africa and the most populous nation in the West Africa. This research considers the bivariate time series analysis of gross domestic product and communication sectors so as to find the effect of communication on the gross domestic product. (GDP). There are other sectors that contributed immensely to the gross domestic product of Nigeria, these include Agriculture, Petroleum products and many others. For example, Petroleum production and export play a dominant role in Nigeria's economy and account for about 90% of her gross earnings. Crude oil/mineral gas has always been the mainstay of the Nigerian economy despite Government's efforts in diversification into agriculture, mining and other sectors. Even though the sector is less than 10% of the country's GDP, it contributes about 65% of Government revenue and 88% of Nigeria's foreign exchange earnings, thereby, having tremendous impacts on various sectors of the economy (Ajayi, 2019). Nigeria has been Africa's largest oil producer for a long time and holds the second largest oil reserves in Africa (after Libya) and the 10th largest oil reserves in the world (NBS, 2010). Nigeria is ranked as the largest producer of crude oil in Africa, and contributes less than 10 percent of Nigeria's gross domestic product (GDP), with approximately 80 percent and 90 percent contributions to the Federal Government's revenue and Nigeria's export earnings respectively (NBS, 2010).

As a measure of performance for an economy, the GDP, or gross domestic product, is the value of all final goods and services produced within the country in a year. GDP data is widely used as economic data in the field of time series modeling and analysis. The data obtained from Gross Domestics Product can be used to meet a wide variety of requirements, such as in industry, finance, research institutions, and other fields. Forecasting economic models is an essential component of the accounting economy's decision-making process. The GDP forecast is necessary for policymakers to forecast the economic model. Time series analysis aims at identifying data patterns and trends as well as explaining data modeling and forecasting. Two principal approaches are adopted to maintain time series analysis, which depends on the time of the frequency domain. Several procedures are used to analyze data within these domains. A useful common technique is the Box-Jenkins ARIMA model, which can be used for univariate or multivariate data set analysis. The ARIMA technique uses moving averages (MA), Smoothing, and Regression methods to detect and remove data autocorrelation. Many statistical tests are used in time series models in order to make them Stationary series and Integrated; thus, Box-Jenkins procedures are used for the determination of ARIMA, and an Ordinary Least Squares method is used to estimate the model parameter. For ARIMA, the AR component represents the effects of previous data observations. The component represents trends, including seasonality. And the MA components represent the effects of previous random shocks (or errors). To fit an ARIMA model into a series, the order of each model component must be selected. Usually, a small integer value (usually 0, 1, or 2) is determined for each component. The quantum development in the communication industry all over the world is very rapid, as one innovation replaces another in a matter of weeks. A major breakthrough is the wireless telephone system, which comes with either fixed wireless lines or the Global System for Mobile Communication (GSM) (Wojuade, 2005). Telecommunication has played a significant role in communication and encourages investment. In respect of employment, Mauaka (2008) found that over 1,000,000 Nigerians have been directly and indirectly employed by the telecommunication operators. Many enterprises and service organizations such as banking, haulage, consultancies, insurance have themselves blossomed. According to Soyinka (2008), mobile phones have empowered the poor by opening up veritable windows of wealth generation for them to get out of the scourge of poverty. According to Adebayo (2008), the introduction of mobile telecoms has the potential to reduce the cost of doing business and increase output. Soyinka (2008) and Ndukwe (2008) revealed that the GSM business has contributed positively to the economy in the area of GSM recharge card printing. Communication is one of the technological sectors that drive and enhance rapid economic growth in Nigeria, thus, changing the lives of the world's population in various forms. It affects many of the government business, corporation organizations, individual transactions, work and general well-being. The development of internationally comparable information statistics is essential for government to be able to adequately design, implement, monitor and evaluate communication policies, (Madueme, 2010). It has over been accepted as one as the main driving force behind organizational competitiveness in the present day business environment. Presently, communication is having dramatic influence on almost all area of human activities and one of the areas of economic activities in which this influence is most manifest is the banking sector. The banking industry is one of the critical sectors of the economy which makes invaluable contributions to the peace of economic growth and development of nations (Ajayi, 2003; Madueme, 2010).

Our main concern as regards this research is the effect of Communication sector to the Gross Domestic Product in Nigeria. However, the study seeks to examine the contribution of Communication on Nigeria's GDP. Most developing nations including Nigeria have embarked on various reforms and programs that foster the use of ICT'S in their economics. These reforms tend to yield little or minimal benefits to economic growth and development, especially when compared with the develop countries of the world. Technological advancement is known to impact fast rate to economic development. In Nigeria, policy on adoption of information and communication technology was initiated in 1999, when the civilian regime came into power of government. The operation of the licensed telecommunication service providers in the country has created some well felt macroeconomic effects in terms of job creation, faster delivery services, reduced transport cost, greater security and higher natural output (Emmanuel and Adebayo, 2011). Attempt to ensure sustainable economic development and poverty reduction of most nations usually involves the development of Agriculture, Mining, Industrial as well as the Service sectors. The industrial revolution of Europe and America, generally and specifically have been presented on technological breakthroughs. The technological advancement in the communication sector in Nigeria with the introduction and installation of GSM system in the country has created much impact in the economic development of the nation. This study will be of great benefit to Government at all levels in Planning, Budgeting and how other sectors of the economy will embrace communication technology in its operation. The outcome of the research will x-ray the contributory effect of the communication sector to the growth of GDP in Nigeria. Also, this study will be of immense benefit to researchers undertaking similar study on communication and gross domestic product of Nigeria. In Nigeria, provision of public infrastructure is grossly inadequate and good communication Services needed for meaningful investment are lacking, and were found to be very costly. Teledensity in Nigeria is still very low. The introduction of the GSM in Nigeria was to expand the Teledensity in the country and to make communication services cheaper and accessible to the common person as it had been introduced in some African countries like South Africa, Ghana, and Benin Republic among others. As a key sector, this research needs to isolate communication from other sectors and x-ray independent contribution of communication to GDP. This is in view of the fact that, being a major sector, communication is expected to make significant contribution to the GDP in Nigeria. This is the focus of the research. In order to achieve the desirable goal of this research in this paper, the research intends to look at the following points, studying the behavior of GDP and Communication for the period under review, fitting of different order of bivariate time series models so as to check the contribution of communication with each suggested model to Nigeria GDP, assessment of the performances of the suggested models and recommend the best model for the macroeconomic variables, and lastly, make a forecast of the future values of GDP using the best estimated time series model.

METHODOLOGY

The method of analysis adopted in this study is the Box - Jenkins (1976) procedure for fitting autoregressive integrated moving average (ARIMA) model. The objectives of Box and Jenkins are to identify the data pattern, fit models and estimate parameters, carry out model diagnostic check and forecast future values of the time series. A general univariate model for ARIMA (p, d, q) process is given as shown below,

$$\varphi_p(B)(1-B)^d X_t = \theta_q(B)\varepsilon_t \qquad (1)$$

Trend Analysis

The trend analysis of the time series data of gross domestic product and communication sector are as shown on figures 3.1, 3.2, 3.5 and 3.6

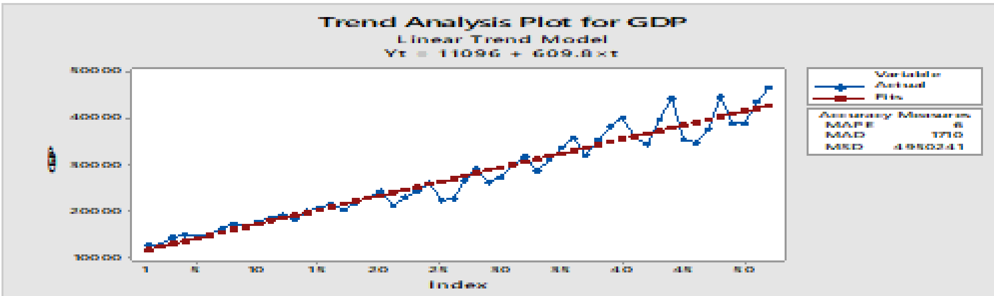

Figure 3.1: Trend Analysis Plot of Gross Domestic Product Time Series data set

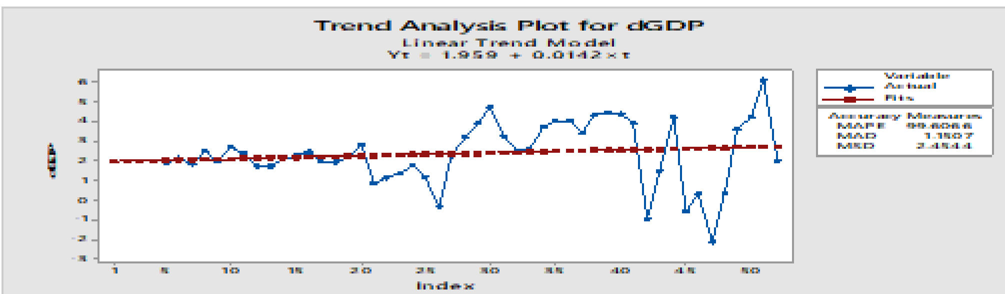

Figure 3.2: Trend Analysis of a Stationary Time Series of a Gross Domestic Product

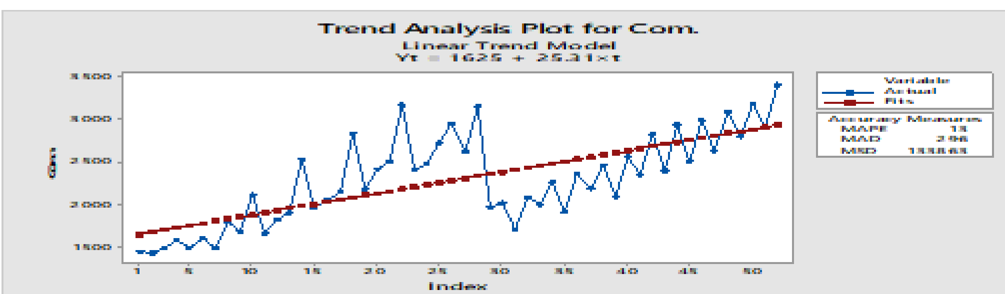

Figure 3.5: Trend Analysis Plot of Communication Sector Time Series data set

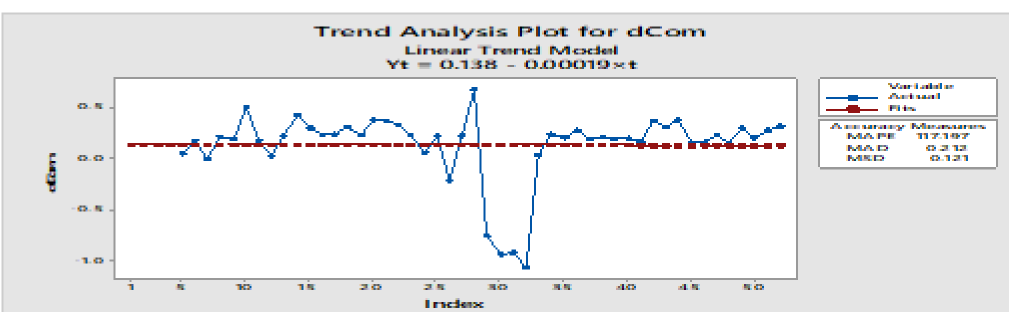

Figure 3.6: Trend Analysis plot of a Stationary Communication Time Series data set

The ACF and PACF of GDP and Communication

The Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) plots displays serial correlation in data that changes over time. It gives a pictorial summary of correlation at different periods of time as shown in figures 3.3, 3.4, 3.7, 3.8 for gross domestic product and communication sector. In figures 3.3, 3.4, 3.7 and 3.8, there are significant cut-off at lag 1 in both the ACF and PACF of the stationary gross domestic product as well as communication sector as shown below.

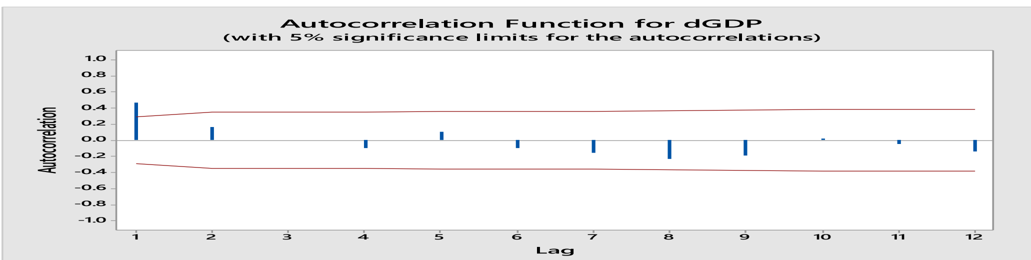

Figure 3.3: The ACF of a Stationary Gross Domestic Product Time Series data set

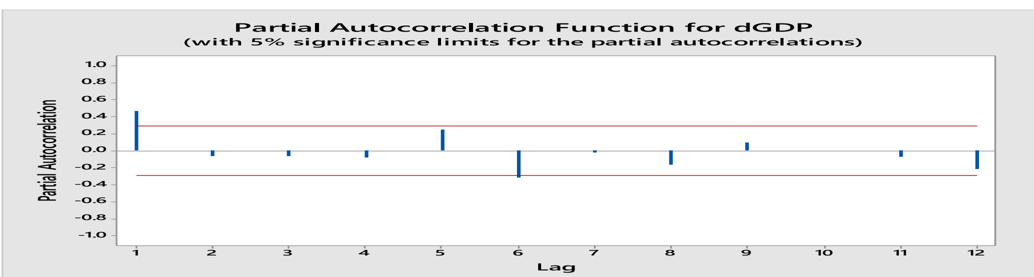

Figure 3.4: The PACF of a Stationary Gross Domestic Product Time Series data set

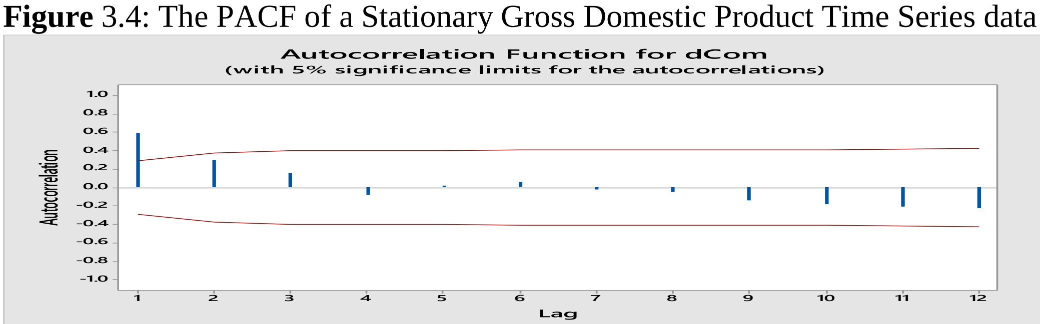

Figure 3.7: The ACF of a Stationary Communication Sector Time Series data set

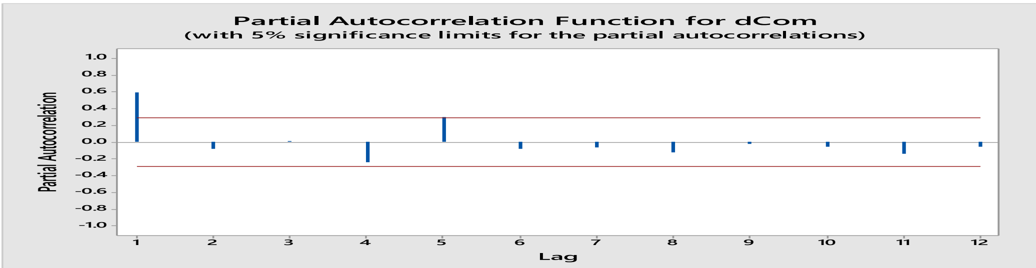

Figure 3.8: The PACF of a Stationary Communication Sector Time Series data set

Thus, from figures 3.3 and 3.4 as well as figures 3.7 and 3.8, the following models are suggested; ARIMA (1 1 1) for Gross Domestic Product, and ARIMA (1, 1, 1) for Communication sector. The variances and the error variances of the ARIMAV, MARDL and MARDL-MA models
used in this work are derived for both the GDP and Communication sector

The ARIMAV Model

Given the ARIMAV Model as shown below,

$$X_{1t} = \varphi_{1.11}X_{1t-1} + \varphi_{1.12}X_{2t-1} - \theta_{1.11}\varepsilon_{1t-1} - \theta_{1.12}\varepsilon_{2t-1} + \varepsilon_{1t} \qquad (2)$$

$$X_{2t} = \varphi_{1.21}X_{1t-1} + \varphi_{1.22}X_{2t-1} - \theta_{1.21}\varepsilon_{1t-1} - \theta_{1.22}\varepsilon_{2t-1} + \varepsilon_{2t} \qquad (3)$$

Where X_{1t} and X_{2t} are the response time series variables representing GDP and Communication sector respectively. $\varphi_{1.11}$, $\varphi_{1.12}$, $\varphi_{1.21}$ and $\varphi_{1.22}$ are autoregressive parameters of contributions of X_{1t} and X_{2t} at lag one to the GDP. $\theta_{1.11}, \theta_{1.12}, \theta_{1.21}, \theta_{1.22}$ are Moving Average parameters of contributions of ε_{1t} *and* ε_{2t} at lag one to the GDP.

Variances of the ARIMAV models

Variances of X_{1t}

From equation (2),

$$E[X_{1t}X_{1t}] = E[(\varphi_{1.11}X_{1t-1} + \varphi_{1.12}X_{2t-1} - \theta_{1.11}\varepsilon_{1t-1} - \theta_{1.12}\varepsilon_{2t-1} + \varepsilon_{1t})(\varphi_{1.11}X_{1t-1} + \varphi_{1.12}X_{2t-1} - \theta_{1.11}\varepsilon_{1t-1} - \theta_{1.12}\varepsilon_{2t-1} + \varepsilon_{1t})]$$

$$E[X_{1t}X_{1t}] = E[\varphi_{1.11}^2X_{1t-1}X_{1t-1} + \varphi_{1.11}\varphi_{1.12}X_{1t-1}X_{2t-1} - \varphi_{1.11}\theta_{1.11}X_{1t-1}\varepsilon_{1t-1} - \varphi_{1.11}\theta_{1.12}X_{1t-1}\varepsilon_{2t-1} + \varphi_{1.11}X_{1t-1}\varepsilon_{1t} + \varphi_{1.11}\varphi_{1.12}X_{1t-1}X_{2t-1} + \varphi_{1.12}^2X_{2t-1}X_{2t-1} - \theta_{1.11}\varphi_{1.12}X_{2t-1}\varepsilon_{1t-1} - \varphi_{1.12}\theta_{1.12}X_{2t-1}\varepsilon_{2t-1} + \varphi_{1.12}X_{2t-1}\varepsilon_{1t} - \varphi_{1.11}\theta_{1.11}X_{1t-1}\varepsilon_{1t-1} - \varphi_{1.12}\theta_{1.11}X_{2t-1}\varepsilon_{1t-1} + \theta_{1.11}^2\varepsilon_{1t-1}^2 - \theta_{1.11}\theta_{1.12}\varepsilon_{1t-1}\varepsilon_{2t-1} - \theta_{1.11}\varepsilon_{1t-1}\varepsilon_{1t} -$$

$\varphi_{1.11}\theta_{1.12}X_{1t-1}\varepsilon_{2t-1} - \varphi_{1.12}\theta_{1.12}X_{2t-1}\varepsilon_{2t-1} + \theta_{1.12}\theta_{1.11}\varepsilon_{1t-1}\varepsilon_{2t-1} + \theta_{1.12}^2\varepsilon_{2t-1}^2 -$
$\theta_{1.12}\varepsilon_{2t-1}\varepsilon_{1t} + \varphi_{1.11}X_{1t-1}\varepsilon_{1t} + \varphi_{1.12}X_{2t-1}\varepsilon_{1t} - \theta_{1.11}\varepsilon_{1t-1}\varepsilon_{1t} - \theta_{1.12}\varepsilon_{2t-1}\varepsilon_{1t} + \varepsilon_{1t}^2]$

$$\gamma_{1t\,1t} = \varphi_{1.11}^2\gamma_{1t\,1t} + 2\varphi_{1.11}\varphi_{1.12}\gamma_{1t\,2t} - 2\varphi_{1.11}\theta_{1.11}\sigma_{et}^2 + \varphi_{1.12}^2\gamma_{2t\,2t} - 2\varphi_{1.12}\theta_{1.12}\sigma_{et}^2 + \theta_{1.11}^2\sigma_{et}^2 + \theta_{1.12}^2\sigma_{et}^2 + \sigma_{et}^2$$

$$\gamma_{1t\,1t}(1-\varphi_{1.11}^2) = \sigma_{et}^2(1+\theta_{1.11}^2+\theta_{1.12}^2 - 2\varphi_{1.11}\theta_{1.11} - 2\varphi_{1.12}\theta_{1.12}) + 2\varphi_{1.11}\varphi_{1.12}\gamma_{1t\,2t} + \varphi_{1.12}^2\gamma_{2t\,2t}$$

$$\gamma_{1t\,1t} = \frac{\sigma_{et}^2(1+\theta_{1.11}^2+\theta_{1.12}^2-2\varphi_{1.11}\theta_{1.11}-2\varphi_{1.12}\theta_{1.12})+2\varphi_{1.11}\varphi_{1.12}\gamma_{1t\,2t}+\varphi_{1.12}^2\gamma_{2t\,2t}}{(1-\varphi_{1.11}^2)} \quad (4)$$

Therefore,

$$\boldsymbol{\sigma_{1et}^2} = \frac{\gamma_{1t\,1t}(1-\varphi_{1.11}^2)-2\varphi_{1.11}\varphi_{1.12}\gamma_{1t\,2t}-\varphi_{1.12}^2\gamma_{2t\,2t}}{(1+\theta_{1.11}^2+\theta_{1.12}^2-2\varphi_{1.11}\theta_{1.11}-2\varphi_{1.12}\theta_{1.12})} \quad (5)$$

Hence, equations (4) and (5) are variances of X_{1t} of ARIMAV model and its error

Variances of X_{2t}

From equation (3),

$E(X_{2t}X_{2t}) = E[(\varphi_{1.21}X_{1t-1} + \varphi_{1.22}X_{2t-1} - \theta_{1.21}e_{1t-1} - \theta_{1.22}e_{2t-1} + e_{2t})(\varphi_{1.21}X_{1t-1} +$
$\varphi_{1.22}X_{2t-1} - \theta_{1.21}e_{1t-1} - \theta_{1.22}e_{2t-1} + e_{2t})]$
$\gamma_{2t2t} = E[(\varphi_{1.21}^2X_{1t-1}X_{1t-1} + \varphi_{1.21}\varphi_{1.22}X_{1t-1}X_{2t-1} - \varphi_{1.21}\theta_{1.21}X_{1t-1}e_{1t-1} -$
$\varphi_{1.21}\theta_{1.22}X_{1t-1}e_{2t-1} + \varphi_{1.21}X_{1t-1}e_{2t} + \varphi_{1.22}\varphi_{1.21}X_{2t-1}X_{1t-1} + \varphi_{1.22}^2X_{2t-1}X_{2t-1} -$
$\varphi_{1.22}\theta_{1.22}X_{2t-1}e_{1t-1} - \varphi_{1.22}\theta_{1.22}X_{2t-1}e_{2t-1} + \varphi_{1.22}X_{2t-1}e_{2t} - \varphi_{1.21}\theta_{1.21}e_{1t-1}X_{1t-1} -$
$\varphi_{1.21}\theta_{1.22}X_{2t-1}e_{1t-1} + \theta_{1.21}^2e_{1t-1}^2 + \theta_{1.21}\theta_{1.22}e_{1t-1}e_{2t-1} - \theta_{1.21}\theta_{1.22}e_{1t-1}e_{2t-1} -$
$\theta_{1.21}e_{1t-1}e_{2t} - \varphi_{1.21}\theta_{1.22}e_{2t-1}X_{1t-1} - \varphi_{1.22}\theta_{1.22}e_{2t-1}X_{2t-1} + \theta_{1.21}\theta_{1.22}e_{2t-1}e_{1t-1} +$
$\theta_{1.22}^2e_{2t-1}^2 - \theta_{1.22}e_{1t-1}e_{2t} + \varphi_{1.21}X_{1t-1}e_{2t} + \varphi_{1.22}X_{2t-1}e_{2t} - \theta_{1.21}e_{1t-1}e_{2t} - \theta_{1.22}e_{2t-1}e_{2t} +$
$e_{2t}^2)]$

$$\gamma_{2t2t} = \varphi_{1.21}^2\gamma_{1t1t} + 2\varphi_{1.21}\varphi_{1.22}\gamma_{1t,2t} - 2\varphi_{1.21}\theta_{1.21}\sigma_{et}^2 + \varphi_{1.22}^2\gamma_{2t2t} - 2\varphi_{1.22}\theta_{1.22}\sigma_{et}^2 + \theta_{1.21}^2\sigma_{et}^2 + \theta_{1.22}^2\sigma_{et}^2 + \sigma_{et}^2$$

$$\gamma_{2t2t} - \varphi_{1.22}^2\gamma_{2t2t} = \sigma_{et}^2 + \theta_{1.21}^2\sigma_{et}^2 + \theta_{1.22}^2\sigma_{et}^2 - 2\varphi_{1.21}\theta_{1.21}\sigma_{et}^2 - 2\varphi_{1.22}\theta_{1.22}\sigma_{et}^2 + \varphi_{1.21}^2\gamma_{1t1t}$$

$$\gamma_{2t2t}(1-\varphi_{1.22}^2) = \sigma_{et}^2(1+\theta_{1.21}^2+\theta_{1.22}^2 - 2\varphi_{1.21}\theta_{1.21} - 2\varphi_{1.22}\theta_{1.22})$$

$$+2\varphi_{1.21}\varphi_{1.22}\gamma_{1t,2t} + \varphi_{1.21}^2\gamma_{1t1t}$$

$$\boldsymbol{\gamma_{2t2t}} = \frac{\sigma_{et}^2(1+\theta_{1.21}^2+\theta_{1.22}^2-2\varphi_{1.21}\theta_{1.21}-2\varphi_{1.22}\theta_{1.22})+2\varphi_{1.21}\varphi_{1.22}\gamma_{1t,2t}+\varphi_{1.21}^2\gamma_{1t1t}}{(1-\varphi_{1.22}^2)} \quad (6)$$

Therefore,

$$\boldsymbol{\sigma_{2et}^2} = \frac{\gamma_{2t2t}(1-\varphi_{1.22}^2)-2\varphi_{1.21}\varphi_{1.22}\gamma_{1t,2t}-\varphi_{1.21}^2\gamma_{1t1t}}{(1+\theta_{1.21}^2+\theta_{1.22}^2-2\varphi_{1.21}\theta_{1.21}-2\varphi_{1.22}\theta_{1.22})} \quad (7)$$

Hence, equations (6) and (7) are variances of X_{2t} ARIMAV model and its error

MARDL Model.

Given the MARDL Model as shown below,

$$X_{1t} = \varphi_{12}X_{2t} + \varphi_{1.11}X_{1t-1} + \varphi_{1.12}X_{2t-1} + \epsilon_{1t} \quad (8)$$

$$X_{2t} = \varphi_{21}X_{1t} + \varphi_{1.21}X_{1t-1} + \varphi_{1.22}X_{2t-1} + \epsilon_{2t} \quad (9)$$

Variables and parameters description are as given above.

Variances of MARDL Models

Variances of X_{1t}

From equation (8)

$$E(X_{1t}X_{1t}) = E[(\varphi_{12}X_{2t} + \varphi_{1.11}X_{1t-1} + \varphi_{1.12}X_{2t-1} + \epsilon_{1t})(\varphi_{12}X_{2t} + \varphi_{1.11}X_{1t-1} + \varphi_{1.12}X_{2t-1} + \epsilon_{1t})]$$

$$\gamma_{1t1t} = E[\varphi_{12}^2X_{2t}X_{2t} + \varphi_{1.11}\varphi_{12}X_{2t}X_{1t-1} + \varphi_{12}\varphi_{1.12}X_{2t}X_{2t-1} + \varphi_{12}X_{2t}\epsilon_{1t} + \varphi_{12}\varphi_{1.11}X_{1t-1}X_{2t} + \varphi_{1.11}^2 X_{1t-1}X_{1t-1} + \varphi_{1.11}\varphi_{1.12}X_{1t-1}X_{2t-1} + \varphi_{1.11}X_{1t-1}\epsilon_{1t} + \varphi_{12}\varphi_{1.12}X_{2t-1}X_{2t} + \varphi_{1.11}\varphi_{1.12}X_{2t-1}X_{1t-1} + \varphi_{1.12}^2X_{2t-1}X_{2t-1} + \varphi_{1.12}X_{2t-1}\epsilon_{1t} + \varphi_{12}X_{2t}\epsilon_{1t} + \varphi_{1.11}X_{1t-1}\epsilon_{1t} + \varphi_{1.12}X_{2t-1}\epsilon_{1t} + \epsilon_{1t}^2]$$

$$\gamma_{1t1t} = \varphi_{12}^2\gamma_{2t2t} + \varphi_{12}\varphi_{1.11}\gamma_{2t\,1t\,(1)} + \varphi_{12}\varphi_{1.12}\gamma_{2t\,2t\,(1)} + \varphi_{1.11}\varphi_{12}\gamma_{2t\,1t\,(1)} + \varphi_{1.11}^2\gamma_{1t1t} + \varphi_{1.11}\varphi_{1.12}\gamma_{1t2t} + \varphi_{1.12}\varphi_{12}\gamma_{2t\,2t\,(1)} + \varphi_{1.12}\varphi_{1.11}\gamma_{2t1t} + \varphi_{1.12}^2\gamma_{2t2t} + \sigma_{1et}^2$$

$$\gamma_{1t1t} = \varphi_{12}^2\gamma_{2t2t} + \varphi_{1.12}^2\gamma_{2t2t} + 2\varphi_{1.12}\varphi_{12}\gamma_{2t\,1t\,(1)} + 2\varphi_{12}\varphi_{1.12}\gamma_{2t\,2t\,(1)} + 2\varphi_{1.11}\varphi_{1.12}\gamma_{2t1t} + \varphi_{1.11}^2\gamma_{1t1t} + \sigma_{1et}^2$$

$$\gamma_{1t1t} - \varphi_{1.11}^2\gamma_{1t1t} = \gamma_{2t2t}(\varphi_{12}^2 + \varphi_{1.12}^2) + 2\varphi_{1.12}\varphi_{12}\big(\gamma_{2t\,1t\,(1)} + \gamma_{2t\,2t\,(1)}\big) + 2\varphi_{1.11}\varphi_{1.12}\gamma_{2t1t} + \sigma_{1et}^2$$

$$\gamma_{1t1t}\,(1 - \varphi_{1.11}^2) = \gamma_{2t2t}(\varphi_{12}^2 + \varphi_{1.12}^2) + 2\varphi_{1.12}\varphi_{12}\big(\gamma_{2t\,1t\,(1)} + \gamma_{2t\,2t\,(1)}\big) + 2\varphi_{1.11}\varphi_{1.12}\gamma_{2t1t} + \sigma_{1et}^2$$

$$\gamma_{1t1t} = \frac{\gamma_{2t2t}(\varphi_{12}^2+\varphi_{1.12}^2)+2\varphi_{1.12}\varphi_{12}\big(\gamma_{2t\,1t\,(1)}+\gamma_{2t\,2t\,(1)}\big)+2\varphi_{1.11}\varphi_{1.12}\gamma_{2t1t}+\sigma_{1et}^2}{(1-\varphi_{1.11}^2)} \quad (10)$$

$$\sigma_{1et}^2 = \gamma_{1t1t}(1 - \varphi_{1.11}^2) - \gamma_{2t2t}(\varphi_{12}^2 + \varphi_{1.12}^2) - 2\varphi_{1.12}\varphi_{12}\big(\gamma_{2t1t\,(1)} + \gamma_{2t2t\,(1)}\big) - 2\varphi_{1,11}\varphi_{1,12}\gamma_{2t1t} \quad (11)$$

Hence equations (10) and (11) are the variances of X_{1t} MARDL model and its error

Variances of X_{2t}
From equation (9)

$$E(X_{2t}X_{2t}) = E[(\varphi_{21}X_{1t} + \varphi_{1.21}X_{1t-1} + \varphi_{1.22}X_{2t-1} + \epsilon_{2t})(\varphi_{21}X_{1t} + \varphi_{1.21}X_{1t-1} + \varphi_{1.22}X_{2t-1} + \epsilon_{2t})]$$

$$\gamma_{2t2t} = E[\varphi_{21}^2X_{1t}X_{1t} + \varphi_{2t}\varphi_{1.22}X_{1t}X_{2t-1} + \varphi_{21}X_{1t}\epsilon_{2t} + \varphi_{1.21}\varphi_{21}X_{1t-1}X_{1t} + \varphi_{1.21}^2X_{1t-1}\epsilon_{2t} + \varphi_{21}\varphi_{1.22}X_{2t-1}X_{1t} + \varphi_{1.21}\varphi_{1.22}X_{1t-1}X_{2t-1} + \varphi_{1.22}^2X_{2t-1}X_{2t-1} + \varphi_{1.22}X_{2t-1}\epsilon_{2t} + \varphi_{21}X_{1t}\epsilon_{2t} + \varphi_{1.21}X_{1t-1}\epsilon_{2t} + \varphi_{1.22}X_{2t-1}\epsilon_{2t} + \epsilon_{2t}^2]$$

$$\gamma_{2t2t} = \varphi^2\gamma_{1t1t} + \varphi_{21}\varphi_{1.21}\gamma_{1t\,1t\,(1)} + \varphi_{21}\varphi_{1.22}\gamma_{1t\,2t\,(1)} + \varphi_{1.21}\varphi_{21}\gamma_{1t\,1t\,(1)} + \varphi_{1.21}^2\gamma_{1t1t} + \varphi_{1.21}\varphi_{1.22}\gamma_{1t2t} + \varphi_{1.22}\varphi_{21}\gamma_{1t\,2t\,(1)} + \varphi_{1.21}\varphi_{1.22}\gamma_{1t2t} + \varphi_{1.22}^2\gamma_{2t2t} + \sigma_{2et}^2$$

$$\gamma_{2t2t} = \varphi_{21}^2\gamma_{1t1t} + \varphi_{1.22}^2\gamma_{2t2t} + 2\varphi_{21}\varphi_{1.21}\gamma_{1t\,1t\,(1)} + 2\varphi_{21}\varphi_{1.22}\gamma_{1t\,2t\,(1)} + 2\varphi_{1.21}\varphi_{1.22}\gamma_{1t2t} + \varphi^2\gamma_{1t1t} + \sigma_{2et}^2$$

$$\gamma_{2t2t} - \varphi_{1.22}^2\gamma_{2t2t} = \gamma_{1t1t}(\varphi_{21}^2 + \varphi_{1.21}^2) + 2\varphi_{21}\varphi_{1.21}\gamma_{1t\,1t\,(1)} + 2\varphi_{21}\varphi_{1.22}\gamma_{1t\,2t\,(1)} + 2\varphi_{1.21}\varphi_{1.22}\gamma_{1t2t} + \sigma_{2et}^2$$

$$\gamma_{2t2t}(1-\varphi_{1.22}^2) = \gamma_{1t1t}(\varphi_{21}^2+\varphi_{1.21}^2) + 2\varphi_{21}\varphi_{1.21}\gamma_{1t\,1t\,(1)} + 2\varphi_{21}\varphi_{1.22}\gamma_{1t\,2t\,(1)} + 2\varphi_{1.21}\varphi_{1.22}\gamma_{1t2t} + \sigma_{2et}^2$$

$$\gamma_{2t2t} = \frac{\gamma_{1t1t}(\varphi_{21}^2+\varphi_{1.21}^2)+2\varphi_{21}\varphi_{1.21}\gamma_{1t\,1t\,(1)}+2\varphi_{21}\varphi_{1.22}\gamma_{1t\,2t\,(1)}+2\varphi_{1.21}\varphi_{1.22}\gamma_{1t2t}+\sigma_{2et}^2}{(1-\varphi_{1.22}^2)} \quad (12)$$

$$\sigma_{2et}^2 = \gamma_{2t2t}(1-\varphi_{1.22}^2) - \gamma_{1t1t}(\varphi_{21}^2+\varphi_{1.21}^2) - 2\varphi_{21}\varphi_{1.21}\gamma_{1t\,1t\,(1)} - 2\varphi_{21}\varphi_{1.22}\gamma_{1t\,2t\,(1)} - 2\varphi_{1.21}\varphi_{1.22}\gamma_{1t2t} \quad (13)$$

Hence equations **(**12) and (13) are the Variances of X_2 MARDL model and its error

MARDL-MA

Given the MARDL-MA Model as shown below,

$$X_{1t} = \varphi_{12}X_{2t} + \varphi_{1.11}X_{1t-1} + \varphi_{1.12}X_{2t-1} - \theta_{1.11}\varepsilon_{1t-1} - \theta_{1.12}\varepsilon_{2t-1} + \varepsilon_{1t} \quad (14)$$

$$X_{2t} = \varphi_{21}X_{1t} + \varphi_{1.21}X_{1t-1} + \varphi_{1.22}X_{2t-1} - \theta_{1.21}\varepsilon_{1t-1} - \theta_{1.22}\varepsilon_{2t-1} + \varepsilon_{2t} \quad (15)$$

Variables are as describe above

Variances of MARDL-MA Models

Variances of X_{1t}

From equation (14)

$$E(X_{1t}X_{1t}) = E[(\varphi_{12}X_{2t} + \varphi_{1.11}X_{1t-1} + \varphi_{1.12}X_{2t-1} - \theta_{1.11}\varepsilon_{1t-1} - \theta_{1.12}\varepsilon_{2t-1} + \varepsilon_{1t})(\varphi_{12}X_{2t} + \varphi_{1.11}X_{1t-1} + \varphi_{1.12}X_{2t-1} - \theta_{1.11}\varepsilon_{1t-1} - \theta_{1.12}\varepsilon_{2t-1} + \varepsilon_{1t})]$$

$$\begin{aligned}\gamma_{1t\,1t} = E[(&\varphi_{12}^2X_{2t}X_{2t} + \varphi_{12}\varphi_{1.11}X_{2t}X_{1t-1} + \varphi_{12}\varphi_{1.12}X_{2t}X_{2t-1} - \varphi_{12}\theta_{1.11}X_{2t}\varepsilon_{1t-1} \\ &- \varphi_{12}\theta_{1.12}X_{2t}\varepsilon_{2t-1} + \varphi_{12}X_{2t}\varepsilon_{1t} + \varphi_{12}\varphi_{1.11}X_{1t-1}X_{2t} + \varphi_{1.11}^2X_{1t-1}X_{1t-1} \\ &+ \varphi_{1.11}\varphi_{1.12}X_{1t-1}X_{2t-1} - \varphi_{1.11}\theta_{1.11}X_{1t-1}\varepsilon_{1t-1} - \varphi_{1.11}\theta_{1.12}X_{1t-1}\varepsilon_{2t-1} \\ &+ \varphi_{1.11}X_{1t-1}\varepsilon_{1t} + \varphi_{12}\varphi_{1.12}X_{2t-1}X_{2t} + \varphi_{1.11}\varphi_{1.12}X_{2t-1}X_{1t-1} \\ &+ \varphi_{1.12}^2X_{2t-1}X_{2t-1} - \theta_{1.11}\varphi_{1.12}X_{2t-1}\varepsilon_{1t-1} - \varphi_{1.12}\theta_{1.12}X_{2t-1}\varepsilon_{2t-1} \\ &+ \varphi_{1.12}X_{2t-1}\varepsilon_{1t} - \varphi_{12}\theta_{1.11}\varepsilon_{1t-1}X_{2t} - \theta_{1.11}\varphi_{1.11}\varepsilon_{1t-1}X_{1t-1} \\ &- \theta_{1.11}\varphi_{1.12}\varepsilon_{1t-1}X_{2t-1} + \theta_{1.11}^2\varepsilon_{1t-1}^2 + \theta_{1.11}\theta_{1.12}\varepsilon_{1t-1}\varepsilon_{2t-1} \\ &- \theta_{1.11}\varepsilon_{1t-1}\varepsilon_{1t} - \varphi_{12}\theta_{1.12}\varepsilon_{2t-1}X_{2t} - \theta_{1.12}\varphi_{1.11}\varepsilon_{2t-1}X_{1t-1} \\ &- \theta_{1.12}\varphi_{1.12}\varepsilon_{2t-1}X_{2t-1} + \theta_{1.11}\theta_{1.12}\varepsilon_{1t-1}\varepsilon_{2t-1} + \theta_{1.12}^2\varepsilon_{2t-1}^2 \\ &- \theta_{1.12}\varepsilon_{2t-1}\varepsilon_{1t} + \varphi_{12}\varepsilon_{1t}X_{2t} + \varepsilon_{1t}X_{1t-1} + \varphi_{1.11}\varepsilon_{1t}\varphi_{1.12}X_{2t-1} \\ &- \theta_{1.11}\varepsilon_{1t}\varepsilon_{1t-1} - \theta_{1.12}\varepsilon_{1t}\varepsilon_{2t-1} + \varepsilon_{1t}^2)]\end{aligned}$$

$$\gamma_{1t\,1t} = \varphi_{12}^2\gamma_{2t\,2t} + \varphi_{12}\varphi_{1.11}\gamma_{2t\,1t\,(1)} + \varphi_{12}\varphi_{1.12}\gamma_{2t\,2t\,(1)} + \varphi_{12}\varphi_{1.11}\gamma_{2t\,1t\,(1)}$$
$$+ \varphi_{1.11}^2\gamma_{1t\,1t} + \varphi_{1.11}\varphi_{1.12}\gamma_{1t\,2t} - \varphi_{1.11}\theta_{1.11}\gamma_{xe} + \varphi_{12}\varphi_{1.12}\gamma_{2t\,2t\,(1)}$$
$$+ \varphi_{1.11}\varphi_{1.12}\gamma_{1t\,2t} + \varphi_{1.12}^2\gamma_{2t\,2t} - \varphi_{1.12}\theta_{1.12}\gamma_{xe} - \varphi_{1.11}\theta_{1.11}\gamma_{xe}$$
$$+ \theta_{1.11}^2\sigma_{et}^2 - \varphi_{1.12}\theta_{1.12}\gamma_{xe} + \theta_{1.12}^2\sigma_{et}^2 + \sigma_{1et}^2$$

$$\gamma_{1t\,1t} - \varphi_{1.11}^2\gamma_{1t\,1t}$$
$$= \varphi_{12}^2\gamma_{2t\,2t} + \varphi_{1.12}^2\gamma_{2t\,2t} + 2\varphi_{12}\varphi_{1.11}\gamma_{2t\,1t\,(1)} + 2\varphi_{12}\varphi_{1.12}\gamma_{2t\,2t\,(1)}$$
$$+ \varphi_{1.11}\varphi_{1.12}\gamma_{1t\,2t} - 2\varphi_{1.11}\theta_{1.11}\sigma_{et}^2 - 2\varphi_{1.12}\theta_{1.12}\sigma_{et}^2 + \theta_{1.11}^2\sigma_{et}^2$$
$$+ \theta_{1.12}^2\sigma_{et}^2 + \sigma_{1et}^2$$

$$\gamma_{1t\,1t}(1 - \varphi_{1.11}^2)$$
$$= \sigma_{1et}^2(1 + \theta_{1.11}^2 + \theta_{1.12}^2 - 2\varphi_{1.11}\theta_{1.11} - 2\varphi_{1.12}\theta_{1.12})$$
$$+ \gamma_{2t\,2t}(\varphi_{12}^2 + \varphi_{1.12}^2) + 2\varphi_{12}\varphi_{1.11}\gamma_{2t\,1t\,(1)} + 2\varphi_{12}\varphi_{1.12}\gamma_{2t\,2t\,(1)}$$
$$+ 2\varphi_{1.11}\varphi_{1.12}\gamma_{1t\,2t}$$

$$\boldsymbol{\gamma_{1t\,1t}} = \frac{\sigma_{1et}^2(1+\theta_{1.11}^2+\theta_{1.12}^2-\varphi_{1.11}\theta_{1.11}-2\varphi_{1.12}\theta_{1.12})+\gamma_{2t\,2t}(\varphi_{12}^2+\varphi_{1.12}^2)+2\varphi_{12}\varphi_{1.11}\gamma_{2t\,1t\,(1)}+2\varphi_{12}\varphi_{1.12}\gamma_{2t\,2t\,(1)}+2\varphi_{1.11}\varphi_{1.12}\gamma_{1t\,2t}}{(1-\varphi_{1.11}^2)} \quad \mathbf{(16)}$$

$$\boldsymbol{\sigma_{1et}^2} = \frac{\gamma_{1t\,1t}(1-\varphi_{1.11}^2)-\gamma_{2t\,2t}(\varphi_{12}^2+\varphi_{1.12}^2)-2\varphi_{12}\varphi_{1.11}\gamma_{2t\,1t\,(1)}-2\varphi_{12}\varphi_{1.12}\gamma_{2t\,2t\,(1)}-2\varphi_{1.11}\varphi_{1.12}\gamma_{1t\,2t}}{(1+\theta_{1.11}^2+\theta_{1.12}^2-2\varphi_{1.11}\theta_{1.11}-2\varphi_{1.12}\theta_{1.12})} \quad \mathbf{(17)}$$

Hence equations (16) and (17) are the Variances of X_{1t} MARDL-MA model and its error

Variances of X_{2t}

From equation (15)

$$E(X_{2t}X_{2t}) = E[(\varphi_{21}X_{1t} + \varphi_{1.21}X_{1t-1} + \varphi_{1.22}X_{2t-1} - \theta_{1.21}\varepsilon_{1t-1} - \theta_{1.22}\varepsilon_{2t-1}$$
$$+ \varepsilon_{2t})(\varphi_{21}X_{1t} + \varphi_{1.21}X_{1t-1} + \varphi_{1.22}X_{2t-1} - \theta_{1.21}\varepsilon_{1t-1} - \theta_{1.22}\varepsilon_{2t-1}$$
$$+ \varepsilon_{2t})]$$

$$\begin{aligned}\gamma_{2t\,2t} = E[(&\varphi_{21}^2 X_{1t}X_{1t} + \varphi_{21}\varphi_{1.21}X_{1t}X_{1t-1} + \varphi_{21}\varphi_{1.22}X_{1t}X_{2t-1} - \varphi_{21}\theta_{1.21}X_{1t}\varepsilon_{1t-1}\\
&- \varphi_{21}\theta_{1.22}X_{1t}\varepsilon_{2t-1} + \varphi_{21}X_{1t}\varepsilon_{2t} + \varphi_{1.21}\varphi_{21}X_{1t-1}X_{1t} + \varphi_{1.21}^2 X_{1t-1}X_{1t-1}\\
&+ \varphi_{1.21}\varphi_{1.22}X_{1t-1}X_{2t-1} - \varphi_{1.21}\theta_{1.21}X_{1t-1}\varepsilon_{1t-1} - \varphi_{1.21}\theta_{1.22}X_{1t-1}\varepsilon_{2t-1}\\
&+ \varphi_{1.21}X_{1t-1}\varepsilon_{2t} + \varphi_{1.21}\varphi_{1.22}X_{1t-1}X_{2t-1} + \varphi_{1.22}^2 X_{2t-1}X_{2t-1}\\
&- \varphi_{1.22}\theta_{1.21}X_{2t-1}\varepsilon_{1t-1} - \varphi_{1.22}\theta_{1.22}X_{2t-1}\varepsilon_{2t-1} + \varphi_{1.22}X_{2t-1}\varepsilon_{2t}\\
&- \varphi_{21}\theta_{1.21}\varepsilon_{1t-1}X_{1t} - \theta_{1.21}\varphi_{1.21}\varepsilon_{1t-1}X_{1t-1} - \theta_{1.21}\varphi_{1.22}\varepsilon_{1t-1}X_{2t-1}\\
&+ \theta_{1.21}^2\varepsilon_{1t-1}^2 + \theta_{1.21}\theta_{1.22}\varepsilon_{1t-1}\varepsilon_{2t-1} - \theta_{1.21}\varepsilon_{1t-1}\varepsilon_{2t} - \theta_{1.22}\varphi_{21}\varepsilon_{2t-1}X_{1t}\\
&- \theta_{1.22}\varphi_{1.21}\varepsilon_{2t-1}X_{1t-1} - \theta_{1.22}\varphi_{1.22}\varepsilon_{2t-1}X_{2t-1} + \theta_{1.22}\theta_{1.21}\varepsilon_{2t-1}\varepsilon_{1t-1}\\
&+ \theta_{1.22}^2\varepsilon_{2t-1}^2 - \theta_{1.22}\varepsilon_{2t-1}\varepsilon_{2t} + \varphi_{21}\varepsilon_{2t}X_{1t} + \varphi_{1.21}X_{1t-1}\varepsilon_{2t}\\
&+ \varphi_{1.22}X_{2t-1}\varepsilon_{2t} - \theta_{1.21}\varepsilon_{1t-1}\varepsilon_{2t} - \theta_{1.22}\varepsilon_{2t-1}\varepsilon_{2t} + \varepsilon_{2t}^2)]\end{aligned}$$

$$\begin{aligned}\gamma_{2t\,2t} = {}&\varphi_{21}^2\gamma_{1t\,1t} + \varphi_{21}\varphi_{1.21}\gamma_{1t\,1t\,(1)} + \varphi_{21}\varphi_{1.22}\gamma_{1t\,2t\,(1)} + \varphi_{21}\varphi_{1.21}\gamma_{1t\,1t\,(1)}\\
&+ \varphi_{1.21}^2\gamma_{1t\,1t} + \varphi_{1.21}\varphi_{1.22}\gamma_{1t\,2t} - \varphi_{1.21}\theta_{1.21}\gamma_{xe} + \varphi_{21}\varphi_{1.22}\gamma_{1t\,2t\,(1)}\\
&+ \varphi_{1.21}\varphi_{1.22}\gamma_{1t\,2t} + \varphi_{1.22}^2\gamma_{2t\,2t} - \varphi_{1.22}\theta_{1.22}\gamma_{xe} - \varphi_{1.21}\theta_{1.21}\gamma_{xe}\\
&+ \theta_{1.21}^2\sigma_{et}^2 - \varphi_{1.22}\theta_{1.22}\gamma_{xe} + \theta_{1.22}^2\sigma_{et}^2 + \sigma_{2et}^2\end{aligned}$$

$$\begin{aligned}\gamma_{2t\,2t} - \varphi_{1.21}^2\gamma_{2t\,2t}\\
= {}&\varphi_{21}^2\gamma_{1t\,1t} + \varphi_{1.21}^2\gamma_{1t\,1t} + 2\varphi_{21}\varphi_{1.21}\gamma_{1t\,1t\,(1)} + 2\varphi_{21}\varphi_{1.22}\gamma_{1t\,2t\,(1)}\\
&+ 2\varphi_{1.21}\varphi_{1.22}\gamma_{1t\,2t} - 2\varphi_{1.21}\theta_{1.21}\sigma_{et}^2 - 2\varphi_{1.22}\theta_{1.22}\sigma_{et}^2 + \theta_{1.21}^2\sigma_{et}^2\\
&+ \theta_{1.22}^2\sigma_{et}^2 + \sigma_{2et}^2\end{aligned}$$

$$\begin{aligned}\gamma_{2t\,2t}(1 - \varphi_{1.21}^2)\\
= {}&\sigma_{2et}^2(1 + \theta_{1.21}^2 + \theta_{1.22}^2 - 2\varphi_{1.21}\theta_{1.21} - 2\varphi_{1.22}\theta_{1.22})\\
&+ \gamma_{1t\,1t}(\varphi_{21}^2 + \varphi_{1.21}^2) + 2\varphi_{21}\varphi_{1.21}\gamma_{1t\,1t\,(1)} + 2\varphi_{21}\varphi_{1.22}\gamma_{1t\,2t\,(1)}\\
&+ 2\varphi_{1.21}\varphi_{1.22}\gamma_{1t\,2t}\end{aligned}$$

$$\gamma_{2t\,2t} = \frac{\sigma_{2et}^2(1+\theta_{1.21}^2+\theta_{1.22}^2-2\varphi_{1.21}\theta_{1.21}-2\varphi_{1.22}\theta_{1.22})+\gamma_{1t\,1t}(\varphi_{21}^2+\varphi_{1.21}^2)+2\varphi_{21}\varphi_{1.21}\gamma_{1t\,1t\,(1)}+2\varphi_{21}\varphi_{1.22}\gamma_{1t\,2t\,(1)}+2\varphi_{1.21}\varphi_{1.22}\gamma_{1t\,2t}}{(1-\varphi_{1.21}^2)} \quad \mathbf{18}$$

$$\sigma_{2et}^2 = \frac{\gamma_{2t\,2t}(1-\varphi_{1.21}^2)-\gamma_{1t\,1t}(\varphi_{21}^2+\varphi_{1.21}^2)-2\varphi_{21}\varphi_{1.21}\gamma_{1t\,1t\,(1)}-2\varphi_{21}\varphi_{1.22}\gamma_{1t\,2t\,(1)}-2\varphi_{1.21}\varphi_{1.22}\gamma_{1t\,2t}}{(1+\theta_{1.21}^2+\theta_{1.22}^2-2\varphi_{1.21}\theta_{1.21}-2\varphi_{1.22}\theta_{1.22})} \quad \mathbf{19}$$

Hence equations (18) and (19) are the Variances of X_{2t} MARDL-MA model and its error

Numerical Verification

ARIMAV Model for X_{1t}

Recall from equation (2)

$X_{1t} = \varphi_{1.11}X_{1t-1} + \varphi_{1.12}X_{2t-1} - \theta_{1.11}\varepsilon_{1t-1} - \theta_{1.12}\varepsilon_{2t-1} + \varepsilon_{1t}$

By adopting Ordinary Least Square method, we have the following estimated model

$$X_{1t} = 0.876X_{1t-1} + 0.75X_{2t-1} - 0.176\varepsilon_{1t-1} - 0.69\varepsilon_{2t-1}$$

The estimated values of X_{1t} shows that X_{1t-1} is significant as shown in tables 4.1 below

Table 4.1 Coefficients Estimates of Parameters

Term	Coef	SE.Coef	T-Value	P-Value
X_{1t-1}	0.876	0.106	8.32	0.000
X_{2t-1}	0.75	1.00	0.75	0.457
ε_{1t-1}	-0.176	0.197	-0.90	0.376
ε_{2t-1}	-0.69	1.29	-0.53	0.596

The error variance of X_{1t} of ARIMAV Model

$$\sigma^2_{1et} = \frac{\gamma_{1t\,1t}(1-\varphi^2_{1.11}) - 2\varphi_{1.11}\varphi_{1.12}\gamma_{1t\,2t} - \varphi^2_{1.12}\gamma_{2t\,2t}}{(1+\theta^2_{1.11}+\theta^2_{1.12} - 2\varphi_{1.11}\theta_{1.11} - 2\varphi_{1.12}\theta_{1.12})}$$

$$\sigma^2_{1et} = \frac{2.54632(1-0.7674) + 2(0.876)(0.75)(0.07489) - 0.5625(0.123670)}{1 + 0.0309 + 0.471 + 2(0.876)(0.176) + 2(0.75)(0.69)}$$

$$\sigma^2_{1et} = \frac{0.62111}{2.8452}$$

$$\sigma^2_{1et} = 0.2183$$

ARIMAV Model for X_{2t}

Recall from equation (3)

$$X_{2t} = \varphi_{1.21}X_{1t-1} + \varphi_{1.22}X_{2t-1} - \theta_{1.21}e_{1t-1} - \theta_{1.22}e_{2t-1} + e_{2t}$$

By adopting Ordinary Least Square method, we have the following estimated model

$$X_{2t} = 0.0093X_{1t-1} + 0.593X_{2t-1} - 0.0110e_{1t-1} + 0.088e_{2t-1}$$

The estimated values of X_{2t} shows that X_{2t-1} is significant as shown in tables 4.3 below;

Table 4.3 Coefficients Estimates of Parameters

Term	Coef	SE.Coef	T-Value	P-Value
X_{1t-1}	0.0093	0.0194	0.48	0.634
X_{2t-1}	0.593	0.185	3.21	0.002
ε_{1t-1}	-0.0110	0.0363	-0.30	0.764
ε_{2t-1}	-0.088	0.237	0.37	0.712

The error variance of X_{2t} of ARIMAV Models

$$\sigma_{2et}^2 = \frac{\gamma_{2t2t}(1 - \varphi_{1.22}^2) - 2\varphi_{1.21}\varphi_{1.22}\gamma_{1t,2t} - \varphi_{1.21}^2\gamma_{1t1t}}{(1 + \theta_{1.21}^2 + \theta_{1.22}^2 - 2\varphi_{1.21}\theta_{1.21} - 2\varphi_{1.22}\theta_{1.22})}$$

$$\sigma_{2et}^2 = \frac{0.123670(1 - 0.351649) + 0.0008260 - 0.00022}{1 + 0.000121 + 0.007744 + 0.00020 - 0.104368}$$

$$\sigma_{2et}^2 = \frac{0.0.8078}{0.903697}$$

$$\sigma_{2et}^2 = 0.0894$$

MARDL Model for X_{1t}

Recall from equation (3.13)

$$X_{1t} = \varphi_{12}X_{2t} + \varphi_{1.11}X_{1t-1} + \varphi_{1.12}X_{2t-1} + \epsilon_{1t}$$

By adopting Ordinary Least Square method, we have the following estimated model

$$X_{1t} = 0.693X_{2t} + 0.8224X_{1t-1} - 0.142X_{2t-1}$$

The estimated values of X_{1t} shows that X_{1t-1} is significant as shown in tables 4.5 below;

Table 4.5 Coefficients Estimates of Parameters

Term	Coef	SE.Coef	T-Value	P-Value
X_{2t}	0.693	0.821	0.84	0.403
X_{1t-1}	0.8224	0.0844	9.74	0.000
X_{2t-1}	-0.142	0.833	-0.17	0.866

The error variance of X_{1t} of MARDL Model

$$\sigma_{1et}^2 = \gamma_{1t1t}(1-\varphi_{1.11}^2) - \gamma_{2t2t}(\varphi_{12}^2 + \varphi_{1.12}^2) - 2\varphi_{1.12}\varphi_{12}\left(\gamma_{2t1t\,(1)} + \gamma_{2t2t\,(1)}\right) - 2\varphi_{1,11}\varphi_{1,12}\gamma_{2t1t}$$

$$\sigma_{1et}^2 = 2.54632(1-0.67634) - 0.123670(0.4802 + 0.0201) - 2 * -0.142 * 0.693 * -0.054088 - 2 * 0.8224 * 0.693 * -0.074890$$

$$\sigma_{1et}^2 = 0.824141 - 0.06181 - 0.01064 + 0.12318$$

$$\sigma_{1et}^2 = 0.874871$$

MARD Model for X_{2t}

Recall from equation (3.14)

$$X_{2t} = \varphi_{21}X_{1t} + \varphi_{1.21}X_{1t-1} + \varphi_{1.22}X_{2t-1} + \epsilon_{1t}$$

By adopting Ordinary Least Square method, we have the following estimated model

$$X_{2t} = 0.023X_{1t} - 0.0137X_{1t-1} + 0.636X_{2t-1}$$

The estimated values of X_{2t} shows that X_{2t-1} is significant as shown in tables 4.7 below;

Table 4.7 Coefficients Estimates of Parameters

Term	Coef	SE.Coef	T-Value	P-Value
X_{1t}	0.0230	0.0273	0.84	0.403
X_{1t-1}	-0.0137	0.0273	-0.56	0.618
X_{2t-1}	0.636	0.118	5.39	0.000

The error variance of X_{2t} of MARDL Models

$$\sigma_{2et}^2 = \gamma_{2t2t}(1-\varphi_{1.22}^2) - \gamma_{1t1t}(\varphi_{21}^2 + \varphi_{1.21}^2) - 2\varphi_{21}\varphi_{1.21}\gamma_{1t\,1t\,(1)} - 2\varphi_{21}\varphi_{1.22}\gamma_{1t\,2t\,(1)} - 2\varphi_{1.21}\varphi_{1.22}\gamma_{1t2t}$$

$$\sigma_{2et}^2 = 0.07364 - 0.001824 + 0.000764 + 0.001071 - 0.00130$$
$$\sigma_{2et}^2 = 0.072351$$

MARDL-MA Model for X_{1t}

Recall from equation (3.19),

$$X_{1t} = \varphi_{12}X_{2t} + \varphi_{1.11}X_{1t-1} + \varphi_{1.12}X_{2t-1} - \theta_{1.11}\varepsilon_{1t-1} - \theta_{1.12}\varepsilon_{2t-1} + \varepsilon_{1t}$$

By adopting Ordinary Least Square method, we have the following estimated model

$$X_{1t} = 0.689X_{2t} + 0.870X_{1t-1} + 0.34X_{2t-1} - 0.169\varepsilon_{1t-1} - 0.75\varepsilon_{2t-1}$$

The estimated values of X_{1t} shows that X_{1t-1} is significant as shown in tables 4.9 below;

Table 4.9 Coefficients Estimates of Parameters

Term	Coef	SE.Coef	T-Value	P-Value
X_{2t}	0.689	0.831	0.83	0.411
X_{1t-1}	0.870	0.106	8.21	0.000
X_{2t-1}	0.34	1.12	0.31	0.761
ε_{1t-1}	-0.169	0.198	-0.85	0.398
ε_{2t-1}	-0.75	1.29	-0.58	0.567

The error variance of X_{1t} of MARDL-MA Models

$$\sigma_{1et}^2 = \frac{\gamma_{1t\,1t}(1-\varphi_{1.11}^2) - \gamma_{2t\,2t}(\varphi_{12}^2 + \varphi_{1.12}^2) - 2\varphi_{12}\varphi_{1.11}\gamma_{2t\,1t\,(1)} - 2\varphi_{12}\varphi_{1.12}\gamma_{2t\,2t\,(1)} - 2\varphi_{1.11}\varphi_{1.12}\gamma_{1t\,2t}}{(1 + \theta_{1.11}^2 + \theta_{1.12}^2 - 2\varphi_{1.11}\theta_{1.11} - 2\varphi_{1.12}\theta_{1.12})}$$

$$\sigma_{1et}^2 = \frac{2.54632(1 - 0.7569) - 0.07300 + 0.15494 - 0.03501 + 0.04430}{1 + 0.02856 + 0.5625 + 0.29406 - 0.51}$$

$$\sigma_{1et}^2 = \frac{0.71024}{1.37512}$$

$$\sigma_{1et}^2 = 0.51649$$

MARDL-MA Model for X_{2t}

Recall from equation (3.20),

$$X_{2t} = \varphi_{21}X_{1t} + \varphi_{1.21}X_{1t-1} + \varphi_{1.22}X_{2t-1} - \theta_{1.21}\varepsilon_{1t-1} - \theta_{1.22}\varepsilon_{2t-1} + \varepsilon_{2t}$$

By adopting Ordinary Least Square method, we have the following estimated model

$$X_{2t} = 0.0234X_{1t} + 0.575X_{1t-1} - 0.0112X_{2t-1} + 0.104\varepsilon_{1t-1} - 0.0068\varepsilon_{2t-1}$$

The estimated values of X_{2t} shows that X_{2t-1} is significant as shown in tables 4.11 below;

Table 4.11 Coefficients Estimates of Parameters

Term	Coef	SE.Coef	T-Value	P-Value
X_{2t}	0.0234	0.0282	0.83	0.411
X_{1t-1}	0.575	0.186	3.09	0.004
X_{2t-1}	-0.0112	0.0315	-0.36	0.723
ε_{1t-1}	0.104	0.239	0.44	0.666
ε_{2t-1}	-0.0068	0.0368	-0.19	0.853

The error variance of X_{1t} of MARDL-MA Models

$$\sigma^2_{2et} = \frac{\gamma_{2t\,2t}\left(1 - \varphi^2_{1.22}\right) - \gamma_{1t\,1t}\left(\varphi^2_{21} + \varphi^2_{1.21}\right) - 2\varphi_{21}\varphi_{1.21}\gamma_{1t\,2t\,(1)} - 2\varphi_{21}\varphi_{1.22}\gamma_{1t\,2t\,(1)} - 2\varphi_{1.21}\varphi_{1.22}\gamma_{1t\,2t}}{\left(1 + \theta^2_{1.21} + \theta^2_{1.22} - 2\varphi_{1.21}\theta_{1.21} - 2\varphi_{1.22}\theta_{1.22}\right)}$$

$$\sigma^2_{2et} = \frac{0.123670(1 - 0.000125) - 0.084392 - 0.69772 - 0.000046 - 0.00058}{1 + 0.010816 + 0.00004626 - 0.1196 - 0.000152}$$

$$\sigma^2_{2et} = \frac{0.08270}{0.8911}$$

$$\sigma^2_{2et} = 0.09280$$

Forecasting

Using ARIMAV Model of GDP to Estimate the value of GDP we have,

$$X_{1t} = 0.876X_{1t-1} + 0.75X_{2t-1} - 0.176\varepsilon_{1t-1} - 0.69\varepsilon_{2t-1}$$

Table 4.12

Observation	GDP	Expected Value
30	4.721	3.456
31	3.218	3.483
32	2.478	2.473

33	2.554	1.729
34	3.689	1.679
42	-0.964	3.468
43	1.492	0.001
45	-0.548	3.324
49	3.566	0.027

Similarly,

Using MARDL Model of Communication to estimate the Value of GDP, we have,

$$X_{2t} = 0.023X_{1t} - 0.0137X_{1t-1} + 0.636X_{2t-1}$$

Table 4.13

Observation	GDP	Expected Value
29	-0.7599	0.4762
31	-0.9277	-0.5899
32	-1.0720	-0.5766
33	0.0311	-0.6565
42	0.3718	0.0311
45	0.1524	0.1733

RESULTS

The research has established interaction and interdependence between the two macroeconomic variables, and has also revealed that each of the variable has contributed significantly to each other at first lag. The error variances of the bivariate time series model were derived for GDP and Communication sector. In comparing the error variances of Gross Domestic Product in ARIMAV, MARDL and MARDL-MA models, it was revealed that the ARIMAV model of Gross Domestic Product has the least variance of 0.2183 making it the best model. Also, comparing the error variances of communication sector in ARIMAV, MARDL and MARDL-MA models, it was revealed that MARDL model has the least error variance of 0.0723, thereby indicating that MARDL model outperformed ARIMAV and MARDL-MA models. Hence, this research has brought to focus the fact that performance of a model over another is predicated upon the nature of the economic data.

CONCLUSION

There is no dispute the fact that every nation has its own unique economic challenges, especially when experiencing ups and downs in some economic sectors. Despite the advocacy and the government's effort towards economic diversification, Nigeria is still bedeviled by a huge reliance on petroleum resources as a major factor in economic growth and sustainability. The importance of crude oil production quantity and price cannot be neglected in view of the role the two macroeconomic variables jointly play in the nation's gross domestic product, but it's important to note also that the communication sector plays a vital role in the nation's GDP; evidence of this is seen in this research work. The research has revealed significant effects of the communication sector on the country's GDP. The two variables have sufficient information to predict the future values of each other. The country's budget proposal for every fiscal year is usually prepared on the basis of the estimated values of the economic data.

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