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CONSTANT RETURNS TO SCALE WITH POLLUTION MODELED AS AN UNPAID FACTOR

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ABSTRACT: *If pollution emissions enter a constant returns to scale production function, the additional output generated by this unpaid factor cannot be an equilibrium phenomenon. Private factors will engage in rent seeking without some form of rationing. Herein, we propose a shared rationing mechanism and investigate the efficiency implications in a setting of devolved decision making. General equilibrium derived optimal conditions show that taxing capital alone will not provide the revenue for efficient local public good levels. Suboptimal public goods provision then leads to inefficient environmental quality. Interestingly, larger shares of emission rents rationed to mobile capital result in environmental competition becoming more fierce. Conversely, when large shares of rent are captured by locally owned fixed-factors, competition for mobile capital is subdued.*

KEYWORDS: environmental federalism, rent dissipation, capital tax competition
JEL CLASSIFICATIONS: H23, H73, H77

INTRODUCTION

Within established environmental economics, governments are charged with the responsibility for much of the desired environmental protection. However, environmental policy making does not emanate from a single authority − rather it is the design of a multilayered public and private effort. In multilevel systems, the interdependence between environmental impacts caused by economic activities across space and over time poses problems regarding the proper assignment of environmental authority. A considerable share of the literature on this 'environmental federalism' is linked to Wallace Oates (see Oates 2002 for a comprehensive review). The general theme throughout his many contributions is that the responsibility of decision making over a specific environmental issue should devolve to the smallest jurisdiction that encompasses nearly all of the benefits and costs associated with it. Accordingly, environmental protection can be tailored to local preferences and production conditions leading to welfare enhancement when compared to a one-size-fits-all standard of environmental protection across all jurisdictions.

Following Oates' tenet, the fundamental responsibility of environmental decision making is the setting of optimal pollution controls. Oates (2002) identifies three benchmark cases impacting effective pollution control standards. First, environmental quality can be thought of as a pure public good − where quality is a function of the *aggregate* level of emissions in a country or beyond. This case refers to pollutants such as greenhouse gases which may vary across jurisdictions but depends on the aggregate, not on local, emissions. In this case, central or international policy making authority is advocated. The second case imagines purely localized emissions. Examples include effects of waste emissions on local water quality, solid waste disposal and ground contamination. This case is described as the strongest candidate for analysis regarding decentralized environmental standard setting. Lastly, most types of pollution cause damage to the source jurisdiction then spillover to neighboring jurisdictions. This case comprises regional pollutants such as vehicle exhausts, some agricultural emissions and river pollutants. These transboundary externalities, in a setting of decentralized authority, generally lead to suboptimal environmental quality. Accordingly, jurisdictional cooperation offers the potential for a more efficient Coasian type of resolution for transboundary effects.

Even within the strongest case for devolved environmental decision making, purely localized pollutants, a vast literature now exists critical of local authority. With local governments free to set their own environmental standards, competition for mobile commerce will likely ensue − resulting in suboptimal levels of pollution. As argued, if local authorities lowered their environmental standards in order to hold down compliance costs for existing and prospective firms, a 'race to the bottom' could be set in motion leading to inefficiently high levels of pollution throughout the economy. The literature on devolved emissions regulation spawned from introducing pollution into the workhorse model of capital tax competition forwarded by Zodrow and Mieszkowski (1986) and Wilson (1986). To date, the literature follows two broad assumptions regarding how emissions are modeled. First, emissions and capital are treated as separate inputs to the production of a final good (Oates and Schwab 1988). Treating emissions as a separate non-purchased factor in a constant returns to scale production function is akin to the classic public input of the 'unpaid' type as described by Feehan (1989) and Oates and Schwab (1991). Cropper and Oates (1992) considered this approach the standard in environmental economics. Second and more recent, the production of a final good is produced with 'dirty' capital (Ogawa and Wildasin 2009). In these more stylized models, emissions are proportional to capital and taxing capital coincides with taxing emissions.[1] Fell and Kaffine (2014) provide a handsome review of the 'dirty' capital literature.

In this article, we opt for the more traditional approach to modeling emissions − pollution as an unpaid factor. Abstracting from public inputs, emission levels included in a constant returns to scale production function is problematic for production efficiency

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[1] The focus of Ogawa and Wildasin (2009) is transboundary pollution fitting Oates' (2002) Benchmark Case 3.

because inclusion gives rise to additional output (rent) that must clear. Oates and Schwab (1988) ration the entirety of these emission rents to local residents. Kunce and Shogren (2005) allow these rents to be chased and captured by a fixed number of explicit mobile firms. Kunce (2022) treats the additional output as a direct in-kind subsidy on jurisdictional capital investment. Herein, we take a different tact by recognizing that such rents will be sought after by *all* economic agents/factors (Henderson 1974). In order to avoid rent seeking factor allocation, a shared rationing mechanism is proposed facilitating emission rent dissipation. What follows is an investigation of efficiency implications, in a devolved setting, of this rationing scheme.

THE MODEL

The economy is made up of a large number of homogeneous jurisdictions − small enough that they behave as price-takers yet large enough that pollution generated in one jurisdiction does not spill over to another.[2] Perfectly competitive polluting firms produce a numeraire private good in each jurisdiction. The production process employs two private inputs − an immobile factor, such as land or labor, and mobile capital. The immobile factor, L, is fixed in supply to local production and is owned entirely by jurisdictional residents. Conversely, the jurisdiction's capital investment, K, may vary due to mobility. In the broader economy, the aggregate capital stock is fixed, $\overline{K}$, hence the model focuses on location choices rather than new capital formation.

In addition to the two private factors, jurisdictional production is impacted by local environmental policy. Following Oates and Schwab (1988), allowed aggregate pollution emissions, E, are treated like an 'unpaid' public input to production. Local authorities set E, which translates to a limitation of the aggregate level of pollution emissions in the jurisdiction. Jurisdictional production exhibits constant returns to scale (CRS) in all inputs and takes the form,

$$F(L,K,E) = F_L L + F_K K + F_E E, \tag{1}$$

where subscripts denote partial derivatives. All marginal products are positive and diminish. Moreover, all factors are technical complements (e.g., $F_{KE} > 0$). Note that including E as an input generates the output contribution, $F_E E$. In equilibrium, this output contribution (profit or rent) must be allocated (clear).

In a more conventional specification, at a return to the locationally fixed-factor, μ, and a return to mobile capital, r, firms will employ the two factors following a cost minimizing objective, the dual. With a homogeneous production function, the technical rate substitution between the two factors is determined solely by the ratio, K/L. Firms then

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[2] As discussed above, this pollution externality fits Oates' (2002) Benchmark Case 2.

choose inputs where the technical rate of substitution equals the ratio of factor prices. Accordingly, the share of total output accruing to capital,

$$S = \frac{rK}{\mu L + rK}, \tag{2}$$

is determined by and is a function of the capital/fixed-factor ratio. Consequently, the fixed factor's share becomes, 1 - *S*. Rationing the contribution to output from allowed emissions, $F_E E$, to productive activity logically follows,

$$SF_E E = SR, \text{ and } (1 - S)F_E E = (1 - S)R. \tag{3}$$

For simplicity, without loss of generality which will become evident, *S* is viewed as parametric herein. Recognizing that *L* is fixed in supply to production, endogeneity of *S* with respect to *K* unnecessarily complicates the exposition that follows.[3] The proper context given above was needed in order to motivate the proposed rationing mechanism. Emissions rent is assumed to increase in both *K* and *E*,

$$R_K = F_{KE} E > 0, \tag{4}$$

$$R_E = F_E + F_{EE} E > 0. \tag{5}$$

In addition to the marginal products, inputs receive their share of emissions rent (*R*) following,

$$\mu = F_L + (1 - S)\frac{R}{L}, \tag{6}$$

$$r + t = F_K + S\frac{R}{K}, \tag{7}$$

where *t* denotes a source-based tax on capital. Capital moves between jurisdictions until all face the same after-tax return, *r*. Under capital tax financing, the public budget constraint becomes,

$$G = tK, \tag{8}$$

where *G* is viewed as Samuelsonian public good. By multiplying both sides of equation (6) by *L*, and multiplying both sides of equation (7) by *K*, production can be re-written as,

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[3] See the Appendix to this paper for an examination of this point.

$$F(L,K,E) = \mu L + (r+t)K\,. \tag{9}$$

Jurisdictional residents are immobile across boarders. They own the local fixed-factor and an equal share of the capital stock, $\overline{K}$, which may be located in other jurisdictions. Jurisdictional income-consumption is equal to,

$$C = \mu L + y\,, \tag{10}$$

where y denotes any exogenous income including returns to capital ownership. Using equation (9), income-consumption becomes,

$$C = F(L,K,E) - (r+t)K + y\,. \tag{11}$$

Residents of a jurisdiction receive utility from consumption and local public goods, but suffer disutility from the level of allowed pollution emissions. Jurisdictional utility takes the form, $U(C,G,E)$, where U_C and $U_G > 0$, but $U_E < 0$. Higher E corresponds to poorer environmental quality where E represents a pure public bad. In keeping with the Arrow-Debreu (Wilson 1999) separation assumption for general equilibrium constructs, residents have two distinct roles in the model. First, as consumers, they seek to maximize utility over a bundle of goods and public services. Second, supplying fixed factor inputs to production and in return receiving income for consumption. More of the mobile factor enhances local production and can provide residents with higher incomes hence more consumption. However, in order to attract the mobile factor, the jurisdiction lowers taxes (effecting the provision of G) and/or relaxes environmental regulations (lowering utility directly) thus setting up a characteristic economic tradeoff.

Benchmark social efficiency requires the maximization of the jurisdictional residents' utility subject to (i) utility in all other jurisdictions is equalized to a fixed level, (ii) aggregate production and consumption clear, and (iii) the mobile capital stock is allocated entirely among jurisdictions (clears). The resulting social optimum conditions from the traditional model are well known (see Oates and Schwab 2002; Wilson 1999) therefore derivation discussion here is keep to a minimum. Social efficiency becomes,

$$\frac{U_G}{U_C} = 1 \quad \forall \text{ jurisdictions}\,, \tag{12}$$

$$\frac{-U_E}{U_C} = F_E \quad \forall \text{ jurisdictions}\,, \tag{13}$$

Equation (12) represents the familiar 'Samuelson condition' for the provision of public goods (Zodrow and Mieszkowski 1986). This optimality condition suggests that the jurisdictional marginal rate of substitution between the public good and consumption equals the marginal cost of providing an incremental increase in the public good. Given equations (8) and (11), the marginal rate of transformation in this context is one for one. Equation (13) shows that jurisdictions should choose a combination of environmental quality and consumption such that the marginal rate of substitution between the two equals the marginal product of emissions (recall that $U_E < 0$). Equation (13) then represents a Samuelson rule for environmental quality (Kunce and Shogren 2005).

The local government's goal is to maximize jurisdictional utility subject to constraint equations (7), (8) and (11). Because the jurisdiction is small relative to the broader economy, it views the net return to capital, r, as fixed. The solution to the government's policy problem is examined by forming the Lagrangean,

$$\begin{aligned}\mathrm{Max}_{C,G,E,K,t}\quad & U(C,G,E)+\lambda_1\left[F_K+S\frac{R}{K}-t-r\right]+\lambda_2\left[tK-G\right]+\\ & \lambda_3\left[F(L,K,E)-(r+t)K+y-C\right]. \qquad (14)\end{aligned}$$

First-order-conditions become,

$$C:\quad U_C-\lambda_3=0, \qquad (15)$$

$$G:\quad U_G-\lambda_2=0, \qquad (16)$$

$$E:\quad U_E+\lambda_1(F_{KE}+S\frac{R_E}{K})+\lambda_3F_E=0, \qquad (17)$$

$$\begin{aligned}K:\quad & \lambda_1(F_{KK}+S\frac{R_K}{K}-S\frac{R}{K^2})+t\lambda_2+\lambda_3(F_K-r-t)=0,\\ & \lambda_1(F_{KK}+S\frac{R_K}{K}-S\frac{R}{K^2})+t\lambda_2-\lambda_3(S\frac{R}{K})=0, \qquad (18)\end{aligned}$$

$$t:\quad -\lambda_1+\lambda_2K-\lambda_3K=0, \qquad (19)$$

where details for the partial derivatives R_K and R_E are found in equations (4) and (5). Solving equations (15) through (19) simultaneously yields optimal rules for the Lagrange multipliers, the capital tax and emissions,

$$\lambda_1=K(U_G-U_C), \qquad (20)$$

$$\lambda_2 = U_G, \tag{21}$$

$$\lambda_3 = U_C, \tag{22}$$

$$t = \frac{SR}{K} + KF_{KK}\left(\frac{1}{U_G \big/ U_C} - 1\right) + SR_K\left(\frac{1}{U_G \big/ U_C} - 1\right), \tag{23}$$

$$\frac{-U_E}{U_C} = F_E + KF_{KE}\left(\frac{U_G}{U_C} - 1\right) + SR_E\left(\frac{U_G}{U_C} - 1\right). \tag{24}$$

ANALYSIS

Starting with equation (20), binding equality constraints force the Lagrange multipliers to be non-zero. Therefore the right-hand-side of equation (20) indicates that local public goods will not be efficiently provided. Recall the social optimum for public goods provision requires that $U_G = U_C$ from equation (12). The multiplier, λ_1, is associated with the capital return constraint and is interpreted as the marginal utility of the capital price when residents' utility is maximized. Necessitating an interior solution, equations (17) and (18) reinforce a *positive* marginal utility of the capital price, hence, public goods will be underprovided (U_G / $U_C > 1$). This result is consistent with those found in Wilson (1986), Zodrow and Mieszkowski (1986) and Oates and Schwab (1988).[4]

A meaningful (interior) solution requires G and therefore t to be positive. The first two terms on the right-hand-side of equation (23) are unambiguously positive − the last term is negative. In most cases at this level of aggregation, the emissions rent term, R, will be sufficiently large insuring $t > 0$. Moreover, the $\lambda_1(\cdot)$ term in equation (18) supports a positive capital tax when, generally, average products of an input decrease in that input (Henderson 1974). What is clear from the optimal conditions is that taxing capital alone will not provide the revenue for efficient local public good levels. Furthermore, jurisdictions' will choose a level of environmental quality following equation (24). With public goods underprovided, the last two terms on the right-hand-side of equation (24) are positive, indicating that the marginal willingness to pay for better environmental quality is greater than F_E , thus jurisdictions' choose more lax standards (higher E). It is important to point out that the source of the environmental inefficiency is directly related to the jurisdictions' tax policy and fiscal structure (Oates and Schwab 1988).

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[4] See the appendix to Oates and Schwab (1988) p. 352.

From equations (23) and (24), removing the distorting incentives (inserting $U_G / U_C = 1$) to underprovide public goods yields the first-best solution, $t^* = SR/K$ and $-U_E / U_C = F_E$. The tax on capital becomes proportional to the emission input's contribution to output per unit of capital employed and environmental standards are set optimally. However, taxing away the proportional emission rents is not enough to efficiently finance the entirety of the public goods structure. Public goods efficient provision must stem from an additional non-distorting tax instrument not modeled above. Without such a tax, t must exceed SR/K , resulting in a deviation from the first-best optimum. To illustrate this, solve equations (23) and (24) simultaneously for $-U_E / U_C$, where,

$$\frac{-U_E}{U_C} = F_E + \frac{(KF_{KK} + SR_K)(KF_{KE} + SR_E)}{(KF_{KK} + SR_K) + \left(t - \dfrac{SR}{K}\right)} - (KF_{KE} + SR_E)\,. \tag{25}$$

When t is not equal to SR/K, first-best is not likely achieved.

Interestingly, equation (25) provides a means to examine how varying S impacts the magnitude of distortions from first-best. In other words, does the exogenous rent rationing rule significantly impact efficiency? In order to see a clear picture, it is useful to consider a specific numerically simulated case. Note that the implications of the right-hand-side of equation (25) can be captured by focusing on production alone. The terms to the right of F_E in equation (25) represent the wedge between decentralized and social environmental efficiency. If this wedge is positive (negative), jurisdictions set allowed emissions higher (lower) than the social optimum (see equation (13)). The impacts of varying t above equality to the emission rents (t^*), can be simulated using a Cobb-Douglas production function of the basic form, $\overline{F} = L^{(1/3)}K^{(1/3)}E^{(1/3)}$, where the output quantity is fixed. Equal input levels and exponents avoid any efficiency distortions stemming from input intensities (recall S is exogenous). Changes in t are modeled in simple percentage increases (e.g., 1%, 5%, 10%) above the numerically simulated value of SR/K. The 'wedge' referred to above will be captured by the percentage difference from social efficiency, F_E. Percentage (proportional) changes are presented instead of raw numerical results because simulated values are relatively meaningless in this context. Three shares of S are considered, 10%, 50% and 90%. Table 1 presents the simulated outcomes.[5]

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[5] Simulations were conducted with Mathematica version 12.1.

Table 1. Simulation Results

***S* = 10%**	
Percent above *t**	**Percent above F_E**
1.0	0.1
5.0	0.3
10.0	0.6
***S* = 50%**	
Percent above *t**	**Percent above F_E**
1.0	0.5
5.0	2.6
10.0	5.5
***S* = 90%**	
Percent above *t**	**Percent above F_E**
1.0	1.6
5.0	8.9
10.0	20.6

Equations (6) and (11) show that increasing *S*, decreases rent returns to the resident owned jurisdictional fixed-factor. As returns to the fixed-factor decrease, local consumption decreases. Recall that jurisdictional residents enjoy positive utility from both private consumption and public goods. Seeking to provide a utility maximizing level of pubic goods in the absence of a non-distorting tax, jurisdictions' must raise *t* above the first-best level. Increasing *t* will have the following effect on capital flow. From equation (7), we define the implicit function,

$$I_1: \; F_K + \frac{SR}{K} - t - r = 0, \tag{26}$$

where,

$$\frac{\partial K}{\partial t} = \frac{-\partial I_1/\partial t}{\partial I_1/\partial K} = \frac{1}{F_{KK} + \frac{SR_K}{K} - \frac{SR}{K^2}} < 0. \tag{27}$$

Recall that the denominator of equation (27) is the average product of capital which decreases in K highlighting that capital is deflected from a jurisdiction that is increasing *t*.

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As capital taxes rise, jurisdictions' possess incentives to relax environmental standards to counteract the capital flight induced by increased taxation. Capital changes, with respect to changes in allowed emissions, following,

$$\frac{\partial K}{\partial E}=\frac{-\partial I_1/\partial E}{\partial I_1/\partial K}=\frac{-(F_{KE}+\frac{SR_E}{K})}{F_{KK}+\frac{SR_K}{K}-\frac{SR}{K^2}}>0. \tag{28}$$

As shown in Table 1, as *t* rises farther away from *t**, environmental standards become more lax. Indeed, the determination of environmental standards and the tax on capital are closely intertwined (Oates and Schwab 1988). Interestingly, as Table 1 depicts, larger shares of emission rents rationed to capital (*S* rising), result in environmental competition becoming more fierce. Conversely, when large shares of rent are captured by local residents, consumption dominates utility. With higher levels of local consumption, the marginal value of *C* declines. Efficient interior solutions for *G* are still conditioned by level curve convexity, but competition for the mobile capital tax base is mitigated.

CONCLUDING REMARKS

Treating pollution emissions as an input, Oates and Schwab (1988) find, in their seminal contribution, that competitive jurisdictions refrain from taxing mobile capital and set efficient emissions levels. Efficiency is achieved, however, by strictly allocating all emission rents and any tax revenues back to immobile residents of a jurisdiction.[6] Herein, efficiency is achieved if non-distorting tax instruments are available to finance local public goods and the capital tax is set equal to rents per capital input. When these conditions do not hold, the well known 'race to the bottom' result is obtained. Moreover, as capital shares an increasing amount of the pollution rents, the race becomes more fierce. Whether emissions behave like a public input of the unpaid type is, of course, an empirical question. For example, Aschauer (1989) provides evidence of the unpaid factor interpretation for certain public capital inputs. Adopting this empirical methodology, using measured emission levels for specific pollutants, could prove to be a fruitful avenue for future research.

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[6] See Oates and Schwab (1988) equation (4) on page 338.

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APPENDIX

Recall the output share equation (2) from the text. Taking the partial derivative of equation (2) with respect to K yields,

$$S_K = \frac{\mu rL}{(\mu L + rK)^2} > 0, \tag{A1}$$

where the capital output share, generally, is increasing in capital. Similarly, take the partial derivative of equation (2) with respect to L,

$$S_L = \frac{-K\mu\, r}{(\mu L + rK)^2} < 0\,, \tag{A2}$$

which shows the capital share decreasing in L. Let's rewrite equation (A1) using equation (A2),

$$S_K = \frac{K\mu\, rL}{K(\mu L + rK)^2} = \frac{-S_L L}{K}\,, \tag{A3}$$

where in the text exposition, L does not vary therefore it is fair to view S as a constant (S_L = 0).

Providing additional support, revisit the right hand side of text equation (7) by taking the partial derivative with respect to K, assuming S is a function of K,

$$\frac{\partial\left[F_K + S\dfrac{R}{K}\right]}{\partial K} = F_{KK} + S\frac{R_K}{K} + \left\{\frac{S_K R}{K}\right\} - S\frac{R}{K^2} < 0\,, \tag{A4}$$

where it is convention to assume that average products decrease in inputs (Henderson 1974). If we assume S is constant, the right-hand-side curly bracketed term in equation (A4) vanishes. Removing this unambiguously positive term from the expression enhances the likelihood of a negative sign and does not change substantive results detailed in the paper (see text equation (18)).