Errors and Misconceptions in Linear Inequalities Among Senior High Students in Mfantseman Municipality

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ABSTRACT: *This study aimed to investigate the different errors and misconceptions made by students when dealing with linear inequalities. The goal was to uncover the nature and causes of these errors and misconceptions among students in Senior High Schools within the Mfantseman Municipality in the Central Region of Ghana. The research employed an explanatory sequential mixed methods design and was conducted in two public Senior High Schools selected from the Municipality. A total of 180 Senior High School students and teachers participated in the study, including 10 teachers. The sample was selected using a random sampling technique, which yielded 170 students from the two chosen public Senior High Schools. Data collection encompassed results from students' tests on linear inequalities, interviews with students, and questionnaires given to mathematics teachers. The collected data was coded and analyzed using descriptive statistics. The study's findings revealed common errors made by students, such as multiplying/dividing by a negative number, substituting inequality symbols with "equal to" symbols, performing operations on only one side or different numbers on the two sides of a compound inequality, as well as errors in algebraic operations, simplification, and arithmetic. Misconceptions observed included confusion between equality and inequality, misconceptions when dividing or multiplying through an inequality by a negative number, and struggles with compound inequalities. Students' difficulties arose from an inadequate understanding of basic inequality concepts, overgeneralization, limited mastery of inequality rules, and insufficient exposure to compound inequalities. Translating word problems into algebraic symbols posed a significant challenge. The study also highlighted that mathematics teachers were aware of the errors made by students. Consequently, teachers made efforts to address these errors during linear* *inequality classes. The findings suggest that teachers not only need assistance in identifying errors but also in understanding how errors can emerge during the learning process. One of the recommendations is to enhance teacher education by emphasizing diverse teacher-student interactions that thoroughly consider students' mathematical ideas. This approach aims to support teachers in effectively utilizing students' experiences in the learning process.*

KEYWORDS: errors, misconceptions, linear, inequalities, knowledge

INTRODUCTION

Inequalities represent one of the most valuable and crucial topics in algebra, yet they remain one of the least comprehended subjects in secondary school Mathematics (Ward, 2016). This topic necessitates students' understanding of other mathematical concepts such as algebra, trigonometry, and analytical geometry (Bicer, Capraro, & Capraro, 2014). Moreover, it plays a vital role in cultivating a conceptual grasp of equality and equations, as inequalities are often regarded as complementary to students' understanding of equality (Tsamir & Almog, 2001).

The examination of students' proficiency in solving algebraic inequalities is not only intriguing due to its mathematical significance but also because it offers an avenue to explore various facets that illuminate their comprehension of algebra. According to the Ministry of Education (2010), Senior High School students are expected to possess the ability to accurately organize, interpret, and present information in written, graphical, and diagrammatic forms. This implies that students should be proficient in interpreting and representing inequalities both in written and graphical formats. However, evidence suggests that students struggle to accurately interpret and represent inequalities, both in written and graphical forms (W.A.E.C, 2011).

Reports from the West African Senior School Certificate Examinations Chief Examiners (WAEC, 2011; 2013; 2015) have highlighted the weaknesses of candidates in inequalities, particularly their difficulty in comprehending the instructions required for solving, interpreting, and graphically representing inequalities. The WAEC chief examiners' reports (2011, 2013, 2015) indicate that when presented with an inequality question, a majority of students were unable to execute tasks such as multiplication, grouping like terms, and illustrating solutions on a number line. This multitude of errors reveals that, among other issues, students struggle with fundamental concepts of inequality.

While numerous studies have investigated students' comprehension of the concept of equality, only limited research has been conducted on students' grasp of the concept of inequality. However, as pointed out by Bicer, Capraro, and Capraro (2014), many students encounter difficulties and misconceptions related to inequalities. Moreover, Halmaghi (2011) discovered that the concept of inequality presents challenges for both high school and university students. Given that each mathematical concept builds upon previous and subsequent concepts, difficulties in mastering a specific concept can lead to struggles in understanding related concepts and contribute to misconceptions. In this context, the concept of inequalities holds significant importance, underscoring the necessity to identify students' misconceptions about inequalities.

The primary objective of this study is to scrutinize the errors and misconceptions that Senior High School students have concerning linear inequalities. Existing research indicates that not only high school students but also pre-service teachers exhibit a weak understanding of this concept, with much of this research conducted beyond the borders of Ghana (El-khateeb, 2016; Bicer, Capraro, & Capraro, 2014; Almog & Ilany, 2012; Halmaghi, 2011). The situation in Ghana mirrors that of other countries, as evident from the W.A.E.C. chief examiners' reports (2011, 2013, 2015). In Ghana, Senior High School students hail from various regions across the country.

Motivated by the scarcity of literature within the scope of the author's research, which revealed no evidence of similar studies in the Mfantseman municipality, the researcher embarked on this study. This study seeks to bridge the gap in knowledge by investigating students' errors and misconceptions regarding inequalities among Senior High School students in the Mfantseman Municipality. In pursuit of this aim, the study addresses the following questions:

1. What errors and misconceptions about linear inequalities do Senior High School students in the Mfantseman Municipality exhibit?
2. What factors contribute to students' errors and misconceptions about linear inequalities in the Mfantseman Municipality?

LITERATURE REVIEW

The Concept of Linear Inequalities

The Canadian Oxford Dictionary defines inequality as the absence of equality among individuals and objects, involving differences in factors such as size, number, quality, and more. This definition provides a foundational understanding of inequalities, especially in a mathematical context. However, for more advanced mathematical work and the progression of this study, a more practical and comprehensive definition is necessary. Furthermore, a comprehensive definition of inequality must encompass and interconnect various discrete elements, including symbols, conventions, and concepts closely associated with the concept of inequality.

Mathematical inequalities, as described by Frempong (2012), are statements in mathematics containing two unequal expressions. Halmaghi (2011) defines algebraic inequalities as mathematical statements asserting that one quantity is either greater than or less than another. In a passage quoted by Botty, Yusof, Shahrill, and Mahadi (2015), Davies and Peck (1855) highlight that the symbols ($<, >, \leq, \geq$ and $\neq$) are used to denote relationships between two unequal algebraic quantities. In these symbols, the first member stands on the left side of the inequality sign, while the second member appears on the right side.

Inequality can be represented as a mathematical statement formed from expressions utilizing symbols such as ($<, >, \leq$ or $\geq$) to compare two quantities (El-khateeb, 2016). Thus, solving the equation ($4 - 2x = 0$) involves finding the value of the variable (x) that satisfies the expression ($4 - 2x = 0$), rendering it equal to zero. On the other hand, the solution to the inequality ($4 - 2x < 0$) encompasses all values of (x) for which the expression ($4 - 2x$) yields a negative value.

Solving inequalities entails determining the value(s) of the variable that fulfill the given order relationship. The nature of the inequality is determined by the use of either the greater-than symbol ($>$) or the less-than symbol ($<$) (Frempong, 2012). For instance, "a is less than b" and "a is greater than b" represent inequalities. Symbolically, "a is less than b" is denoted as $a < b$, while "a is greater than b" is denoted as $a > b$. Similarly, "a is less than or equal to b" is denoted as $a \leq b$, "a is greater than or equal to b" is denoted as $a > b$, and "a is not equal to b" is represented as $a \neq b$ (Frempong, 2012). In brief, an inequality is established whenever two expressions are linked using one of the five symbols: $<, >, \leq, \geq, \neq$.

Student's Errors and Misconceptions on Linear Inequalities

While students might possess commendable mathematics grades and demonstrate proficiency in tackling textbook questions and exam problems, their grasp of the foundational principles underlying linear inequalities often remains uncertain. According to the Ministry of Education (2010), it is expected that Senior High School students should not only be capable of elucidating inequalities using mathematical symbols, but also comprehending their significance by deciphering the solutions these inequalities offer. Nevertheless, a range of scholars (Almog & Ilany, 2012; Vaiyavutjamai & Clements, 2006; Tsamir & Bazzini, 2004) have observed that a significant number of high school students harbor misconceptions and encounter challenges that lead to a misconstrued understanding of inequalities. Consequently, this hampers their ability to accurately solve equations and correctly interpret their implications.

Over the course of the past few decades, researchers in the realms of mathematics education and educational psychology have pinpointed several prevalent misconceptions that students frequently harbor concerning inequality concepts. Although the following is not an exhaustive compilation, this discussion will delve into a few extensively studied misconceptions, encompassing the tendencies to treat inequalities as equalities, grapple with difficulties in comprehending solutions to inequalities, inaccurately represent inequalities on a numerical scale or graph, and improperly apply the rules governing inequalities.

Treating Inequalities as Equations

Throughout the literature, a recurring argument is that students often confuse inequalities with equalities (Halmaghi, 2011; Davis & Gripper, 2012; Davis, 2013; Bicer et al., 2014; Rushton, 2018). This misconception arises from the belief that inequalities and equalities follow the same mathematical solution process. Consequently, students approach inequality problems in the same way as equations, applying memorized transformations instead of comprehending the concepts of equivalence and order. They are typically taught to solve inequalities using procedures similar to solving equations, with the added rule that the sign should be switched when multiplying or dividing by a negative number (Vaiyavutjamai & Clements, 2006).

Prestege and Perks (2005) conducted a study involving prospective teachers, revealing that students often treat inequalities as equations and merely restore the sign once they solve the equation. For instance, when dealing with the inequality $-65x^3 > 0$, they might solve it as if it were the equation $-65x^3 = 0$, leading to the incorrect conclusion that $x^3 = 0$, and subsequently $x = 0$. In reality, the solution should be x < 0, as the rule regarding the change in the direction of the inequality when multiplying or dividing by a negative number is frequently forgotten (Bicer et al., 2014)**.**

Bazzini and Tsamir (2003) interviewed sixteen to seventeen-year-old Italian and Israeli students, finding that these students often solve inequalities using equations as a prototype model, following the principle of "applying the same operation with the same numbers on both sides." Another study by Tsamir and Bazzini (2004) explored student solutions to inequalities resulting in a single value (e.g., $5x^4 \leq 0 \rightarrow x = 0$ Students in this study tended to reject single values as valid solutions to inequalities due to two intuitive beliefs: firstly, that inequalities yield inequalities, and secondly, that solving inequalities is akin to solving equations. Davies and Gripper (2012) and Davies (2013) investigated the treatment of linear inequalities in Grade 10 students' solutions and textbooks in South Africa, highlighting that the textbook's "rules for solving inequalities" primarily emphasized the spatial orientation of the inequality symbol, such as the rule that "multiplying or dividing both sides of an inequality by a negative number reverses the inequality sign."

While many sources concur that the equation/inequality connection contributes significantly to students' struggles with solving inequalities, Kieran (2004) suggested that this connection could potentially be exploited for educational purposes. Kieran's study of eighth-grade Japanese students introduced inequalities through contextual problems, revealing that students initially approached the problems as equations and then adapted the equation's solution to find the inequality's solution.

Kieran proposed that a close relationship exists between equality and inequality, presenting an opportunity to help students utilize this connection beneficially while avoiding its pitfalls.

Garuti et al. (2001) noted that students' confusion between equations and inequalities could stem from how inequalities are predominantly taught as algorithmic processes in many countries, often treated as subordinate topics related to equations. For example, in countries like Italy, Israel, and France, students tend to approach inequalities algorithmically, applying algebraic transformations suitable for equations but overlooking inequalities' distinct properties. In South Africa, Davis (2013) observed that inequalities are taught as a sub-topic of equations, and students are explicitly instructed to solve inequalities using algebraic manipulations, following similar procedures as for equations, but with exceptions such as reversing the inequality sign when multiplying or dividing by a negative number. This approach overlooks the concept of equivalence in equations and the notion of numerical order in inequalities (Davis, 2013).

Tsamir, Almog and Tirosh (1998), found that students often treated inequalities as equations without considering the differences in meaning conveyed by different symbols. Some students even substituted the inequality sign with an equal sign and solved the resulting equation. This disconnect led them to focus solely on the procedural aspects of algebraic expressions, dissociated from their underlying meanings (Tsamir et al., 1998). Lim (2006) argued that an excessive emphasis on procedural rules in algebra had marginalized the semantic and structural aspects of the subject, overlooking the deeper understanding it requires.

Several studies propose that students solve inequalities without truly grasping the concept of inequality. Bazzini and Tsamir (2001) investigated students' responses to both standard and non-standard inequality tasks. They discovered that students often rely on learned procedures rather than a profound mathematical understanding when solving inequalities, emphasizing the need to consider algebra as more than just formal manipulation. Bazzini and Tsamir (2003) aligned these findings with Fischbein's (1993) theory of formal, intuitive, and algorithmic knowledge in mathematical thinking. They argued that students tend to use intuitive and algorithmic knowledge when solving inequalities, lacking a more comprehensive theoretical understanding that encompasses propositional thinking (Bazzini & Tsamir, 2003).

Interpretations of Inequalities

When students solve inequalities, they frequently lack a comprehensive understanding of the implications of their solutions (Bicer et al., 2014). Vaiyavutjamai and Clements (2006) observed that students who treat inequalities as equations might arrive at correct answers, yet they struggle to verify the accuracy of their results. Tsamir and Bazzini (2004) uncovered a common misconception among students that "solutions of inequalities must be inequalities" (p. 807). Furthermore, Vaiyavutjamai and Clements (2006) highlighted that certain students mistakenly assume that only a singular value can satisfy an inequality, and they tend to believe that solutions to inequalities are limited to single points rather than intervals or infinite sets. These misunderstandings contribute to students' challenges in comprehending the outcomes of inequalities.

Another mathematical misconception is overspecialization, wherein students inappropriately confine a specific scenario to broader cases (Egodawatte, 2011). Tsamir and Bazzini (2004) surveyed 148 high-school students in Israel about their grasp of inequalities, leading to the conclusion that many students incorrectly assume that the outcomes of inequalities must exclusively be inequalities. However, solutions to inequalities can span from individual values to entire sets of numbers (Almog & Ilany, 2012). For instance, when considering an integer value of x where, $3 < x < 5$ only one value (4) fulfills this condition, exemplifying a singular numeric solution. Conversely, if x is a real number and $3 < x < 5$, there exist infinitely many real numbers between 3 and 5 that satisfy this inequality. Surprisingly, Tsamir and Bazzini (2004) discovered that numerous high-school students believed that only a solitary value can validate an inequality, even when dealing with an infinite solution set.

Vaiyavutjamai and Clements (2006) investigated the comprehension of linear equations and inequalities among 31 secondary school students. Their research revealed that some students, despite deriving accurate solutions to inequalities, tended to provide only a single value as their response. For instance, when faced with the inequality $6x \geq 6$ and considering x as an integer, even students who correctly deduced that $x \geq 1$ opted to write down solely "1" as the solution. Following the examination, Vaiyavutjamai and Clements (2006) conducted interviews to gain insights into students' problem-solving processes. The interview responses mirrored the test outcomes, underscoring that students erroneously believe that only one value can satisfy an inequality.

Factors Contributing to Students' Errors and Misconceptions in Inequalities

Errors and misconceptions often arise in students due to a variety of sources. These misconceptions stand in contrast to established mathematical and scientific concepts. Many researchers have highlighted several contributing factors to these misconceptions, including influence from everyday life experiences (Suniati, Sadia & Suhandana, 2013; Widarti, Permanasari, & Mulyani, 2016), the role of teachers (Erman, 2017; Gudyanga & Madambi, 2014), and the potential confusion caused by the everyday language used (Erman, 2017; Suniati, Sadia & Suhandana, 2013).

El-Shara' and Al-Abed (2010) observed that the category of Common Mistakes made by students can be attributed to three primary sources: the nature of the subject matter, the individual student's characteristics, and the teacher. The teacher holds the responsibility of mitigating the impact of each source of misconception. Three major factors contribute to students' errors and misconceptions in mathematics learning: the influence of the teacher, the language employed, and the personal experiences of the students.

METHOD

Research Design

This study utilized the explanatory sequential mixed methods research design to examine the errors and misconceptions related to linear inequalities among senior high school students in the field of mathematics. As outlined by Creswell (2014), the explanatory sequential mixed methods research design encompasses a two-fold approach, wherein the initial phase involves the collection and analysis of quantitative data. Subsequently, the outcomes of this analysis are employed to inform and enhance the second phase, which involves qualitative research.

Participants

A straightforward random sampling technique was employed to choose a total of one hundred and seventy (170) students from two senior high and technical schools located within the Mfantseman municipality in the Central Region of Ghana. The rationale behind this choice was that by the second year, senior high school students would have encountered a comprehensive range of inequalities as outlined in the syllabus. This exposure would have facilitated the delineation of the extent of their comprehension of the subject matter. As a result, it was anticipated that these students would possess the capacity to effectively solve, interpret, or represent any variant of linear inequality.Additionally, a purposive sampling method was adopted to deliberately select twelve students from the larger pool of 170 students. This selected group was then interviewed, with the intention of garnering more detailed insights.

Research Instruments

The data collection process in this study involved the utilization of a knowledge test and an interview guide focused on linear inequalities. The structured knowledge test was designed to gauge the students' proficiency in linear inequalities and to identify the specific types of errors they tended to make. Subsequently, interviews were conducted to complement the test results.

The linear inequality test comprised two sections. The first section encompassed ten multiple-choice questions, while the second section comprised five open-ended questions. Each participant was allotted one hour to complete the test.

Simultaneously, an interview guide was prepared, consisting of eight fundamental questions along with corresponding follow-up questions. This guide was intended to elicit in-depth insights into the thought processes of the students, particularly regarding their responses to the test questions.

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Data Analysis

The Linear Inequalities Knowledge test encompassed the quantitative data for the study. Following the administration of the test, the answer sheets were assessed, and the errors made by students were categorized and assigned corresponding numerical values. This dataset was meticulously input into the Statistical Package for Social Sciences (SPSS) version 20, and subsequently examined using descriptive statistical methods.

The information obtained via the interviews was collected and subjected to qualitative analysis, serving as a complementary source of data alongside the test results. The interviews were recorded and subsequently transcribed verbatim for accurate representation.

RESULTS AND DISCUSSION

Senior High School Students' Errors and Misconceptions on Linear Inequalities

In order to explore students' errors and misconceptions regarding linear inequalities, a written test comprising five questions related to linear inequalities was administered to the participants. Any responses that were deemed incorrect were aggregated, and from these, coherent groupings of errors or potential misconceptions were established, creating a systematic framework for categorizing errors or misconceptions.

Subsequently, the identified error categories underwent a thorough review, aiming to merge similar groups and differentiate distinct categories where applicable. Percentages pertaining to each error type were then meticulously calculated, utilizing the number of students who provided answers for each respective question (as presented in Table 4.1).

Table 4.1: Frequencies and Percentages of Errors Committed by Students

No.	Error type	Frequencies	Percentage
1.	Not reversing / (Reversing when not supposed to) the inequality sign	86	50.9%
2.	Replacing inequality symbols with "equal to" symbol	40	23.7%
3.	Errors in the distributive property	28	16.6%

4.	Subtracting or adding to only one side/ different numbers from the two sides of a double Linear inequality	55	32.5%
5.	Reducing double linear inequality to a single inequality/ ignoring one side of a double linear inequality.	28	16.6%
6.	Wrongful transformation of word problems into mathematical statements (inequalities)	72	42.6%
7.	Incorrect representation of solutions on a number line	40	23.7%
8.	Errors in algebraic operations, simplifications and arithmetic	117	69.2%
9.	Wrongful separation of double inequality	17	10.1%
10.	Problem with the presentation of solutions	36	21.3%

The study revealed that a substantial portion of the participants, namely 69.2%, exhibited errors in algebraic operations, simplifications, and arithmetic. Additionally, a notable 50.9% of the participants erred when performing multiplication or division involving negative numbers within inequalities. Among the surveyed individuals, a significant 42.6% struggled to translate word problems into mathematical expressions (inequalities), while 32.5% committed the error of either shifting only one number or different numbers to the two sides of a double inequality. Furthermore, 23.7% of participants mistakenly replaced inequality symbols with the "equal to" sign, while an equal percentage (23.7%) encountered difficulties when representing their solutions on a number line. Errors in accurately presenting the solution set were made by 21.3% of participants, and 16.6% demonstrated misunderstandings related to the distributive property.

Moreover, an additional 16.6% and 10.1% of participants made the subsequent errors: reducing a double inequality to a single one, either by neglecting one side or transferring all terms from one side to the other. Additionally, some participants erroneously separated the double inequality while attempting to ascertain the solution to the problem.

The findings concerning the learners' errors are in line with the outcomes of prior research, as observed in the works of El-khateeb (2016), Bicer, Capraro, and Capraro (2014), Egodawatte (2011) and Tsamir and Almog (2001). Within this study, the range of errors displayed by learners was diverse. The analysis indicated that learners had a limited grasp of the inequality concept, often solving equations instead of inequalities. Challenges were also noted in relation to the order of operations within inequalities, as well as misconceptions surrounding the multiplication/division of inequalities by non-necessarily positive factors. Errors encompassed aspects such as the distributive property, algebraic manipulations, simplifications, and arithmetic. Furthermore, complications emerged from dealing with compound inequalities, converting word problems into mathematical expressions, and effectively representing solutions on a number line.

Causes of Students' Errors and Misconceptions

This section's objective is to discern the underlying reasons behind students' errors and misconceptions concerning linear inequalities, utilizing insights garnered from interviews. The interviews conducted with students unveiled several factors that contribute to their errors and misconceptions:

Inadequate Mastery of Inequality Rules: Insufficient familiarity with the rules governing inequalities results in the improper alteration of inequality directions, even when dividing inequalities by positive numbers.

Lack of Explanation Regarding Sign Changes: Some students indicated that teachers fail to explain the rationale behind the change in direction of the inequality sign when dividing by a negative number.

Perception of Inequality Symbol: Certain students perceive the inequality symbol not as a representation of a mathematical relationship, but rather as a mere separator between two distinct groups.

Limited Understanding of Inequality Meaning: Some students struggle with understanding or comprehending the meaning underlying inequalities.

Overgeneralization of Concepts: An inclination to overgeneralize concepts contributes to errors and misconceptions.

Limited Exposure to Compound Inequalities: Inadequate exposure to compound inequalities adds to students' challenges in this area.

Student Carelessness: Errors stemming from student carelessness were also identified as a contributing factor.

Insufficient Grasp of Algebraic Processes: A limited understanding of algebraic processes plays a role in students' errors.

Students Attitudes Towards Mathematics: Individual attitudes toward mathematics can influence the occurrence of errors and misconceptions.

The outlined causes of students' errors and misconceptions align with findings from prior research. For instance, Mamba (2012) emphasizes that students in the early stages of learning about inequalities often grapple with misconceptions regarding the meaning of the inequality sign. He advocates for perceiving the inequality sign as a relational symbol rather than a mere connector between two sides.

Additionally, the issue of insufficient explanation by teachers regarding the alteration of inequality signs when dividing by negative numbers resonates with viewpoints presented by Mamba (2012), Ciltas, Alper, and Tatar (2011), and Ciltas, Isik, and Kar (2010). This factor consistently emerges as a significant contributor to students' errors, both in incorrectly reversing and failing to reverse the inequality sign when required.

CONCLUSION

The study conclusively illustrated that students frequently commit errors in the realm of linear inequalities, and a subset of these errors can be categorized as misconceptions. Notably, compound inequalities emerged as a particularly vulnerable area, prone to generating a higher number of mistakes. This investigation further shed light on the root causes underpinning these errors and misconceptions.

At the Senior High school level, educators wield a pivotal influence in establishing a robust foundational comprehension, a factor that subsequently contributes to students' success in the domain of mathematics. The recognition of prevalent errors and misconceptions, coupled with a deep understanding of their origins, should be integrated into the curriculum of initial teacher training. Such an approach would empower teachers with the requisite awareness and knowledge. Consequently, this proactive measure holds the potential to curtail the frequency of these errors and misconceptions, resonating with the findings of Schnepper and McCoy (2013). Their research demonstrated that diagnosing these errors and misconceptions not only facilitated the swift assimilation of new knowledge but also prolonged retention among students.

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