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Development of a Distribution Free Multivariate Canonical Analysis

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ABSTRACT: *This study illustrates the use of Median Polish analysis (MP) as a distribution free procedure that can be used to identify multivariate canonical data structures. The MP may be especially useful in situations where the sample sizes are small, or where the distributions do not meet the assumptions of conventional Canonical Correlation analysis (CC). We begin by comparing the CC and MP analyses with a sample multivariate data set. We go on to compare Type 1 error rates for each of these analyses using Monte Carlo procedures in which we manipulated sample size and skewness of the data distributions. Results indicated that Type 1 error was significantly higher for the CC relative to the MP when the distributions were skewed and/or when the sample sizes were n=20 or 30.*

KEYWORDS: median polish analysis, canonical correlation analysis, exploratory research, hypothesis testing, Monte Carlo simulation, multivariate, distribution free, non-parametric

INTRODUCTION

Canonical statistical models summarize the relationship between a set of multiple predictors and a set of multiple outcomes (https://en.wikipedia.org/wiki/Canonical_correlation). The canonical correlation (CC-R) is a measure of this overall associative relationship between the sets (Knapp, 1978). The CC also provides weights that summarize the structure of the relationship, specifically, the extent to which each predictor or outcome variable in the model contributes to the shared relationship between the variable sets. The goal of a conventional CC is first, to identify the best fitting relationship between a set of predictors and a set of outcomes and second, to interpret this relationship by determining which of the predictors and outcomes are most sensitive to the overall canonical relationship between the sets. This modeling procedure can be diagrammed as follows (Harris, 2013; Tofallis, 1999)

Predictors	Outcomes
□P1+□P2 + □P3 + □P4 =	□O1 + □O2+ □O4 + □O5

Table 1. Basic Canonical Correlation Model

To illustrate this process, Table 1 displays an adaptation of the generalized linear model for canonical analysis. The hypothetical model has a set of four predictors and a set of four outcomes. The P variables on the left are the predictors and the O variables on the right are the outcomes. The CC-R assesses the overall correlation between the Predictors and Outcomes (P and O). The CC-R is a Pearson Product Moment correlation between the weighted linear combinations of the P and O variables. The CC also includes beta values ("□") that are weighting coefficients that depict the relative contribution of each variable to the overall correlation between the sets. The mathematics of CC ensures that no other weights applied to the predictors and outcomes could create a larger overall CC-R (Pedhauser, 1982).

LITURATURE AND THEORETICAL UNDERPINNINGS

The CC is applicable to a variety of research designs. For example, reducing the outcome set to one variable changes the CC into the familiar multiple regression model (Pedhazur, 1982). Shrinking the predictors and outcomes to one variable each converts the CC to a bivariate correlation. The model can be used to test hypotheses regarding theoretical relationships that may exist between the predictors and the outcomes. The CC can also be used for exploratory research and for data mining operations (Wendt, 1979). Although the CC procedure assumes that the variables are continuous, it may also be appropriate for analyzing contingency tables (Holland, Levi, & Watson, 1980); Isaac & Milligan, 1983; Kaiser & Cerny, 1980).

A common problem with CC, or any other parametric analysis, is that the data may not conform to the assumptions of the statistical test (Kaiser & Cerny, 1980). Even though violations of assumptions may not be a major problem in any one experiment, the problem is compounded with each experiment for which the assumptions are not met. Although there are a variety of non-parametric *univariate* analyses that may be substituted in situations where there are gross violations of normality, there are few *multivariate* non-parametric alternative analyses that can be used for hypothesis testing or exploratory data analysis (Michaeli, Wang and Livescu, K. (2016).

The purpose of this research is to illustrate the use of a well-developed exploratory data analysis procedure (Median Polish analysis, Tukey, 1977) as an alternative to conventional CC in situations where the data may be skewed or when the sample sizes are small. We begin by providing a brief overview of the computational mechanics of the CC and MP (Klawonn, Balasubramanian, Crull, Kukita, & Pressler, (2013). We then illustrate how each analysis would be applied and interpreted with the same sample data set. We end with a Monte Carlo simulation to assess the stability of the CC and MP test statistics when the data are skewed or with variations of sample size.

METHODOLOGY

The mechanics and interpretation of CC

Pedhauser, (1982, pps, 722-748) presents a thorough description of the computational procedures that underlie the CC. The conventional CC derives from a correlation matrix that illustrates the bivariate relationship between each variable and every other variable (see Table 2). To illustrate, we collected sample data from ten students regarding their academic and employment history which is described below. We then fit a CC model to these data to illustrate how a conventional parametric CC would proceed. The goal of the CC was to assess the relationship between self-report measures of acquired skills in high school and subsequent achievement measures in college and beyond. Each of these participants had completed high school and had earned a BS degree. Each had been working full time for at least five years after graduation. These data included self-report of high school High School GPA (HSGPA), SAT Verbal scores, (SATV), SAT Math scores (SATM) and a measure of finger dexterity (DEXT) as predictors of Freshman GPA (GPAF), Senior GPA (GPAS), typing speed (TYPE) and Salary (SALARY) as outcomes.

	HSGPA	*SATV*	*SATM*	*DEXT*	*GPAF*	*GPAS*	*TYPE*	*SALARY*
HSGPA	*1*	*0.978*	*0.939*	*0.286*	*0.946*	*0.961*	*0.299*	*0.873*
SATV	*0.978*	*1*	*0.916*	*0.356*	*0.964*	*0.975*	*0.383*	*0.850*
SATM	*0.939*	*0.916*	*1*	*0.243*	*0.908*	*0.899*	*0.565*	*0.768*
DEXT	*0.286*	*0.356*	*0.243*	*1*	*0.428*	*0.31*	*0.385*	*0.401*
GPAF	*0.946*	*0.964*	*0.908*	*0.428*	*1*	*0.983*	*0.445*	*0.751*
GPAS	*0.961*	*0.975*	*0.899*	*0.31*	*0.983*	*1*	*0.326*	*0.777*
TYPE	*0.299*	*0.383*	*0.282*	*0.985*	*0.445*	*0.326*	*1*	*0.392*
SALARY	*0.873*	*0.85*	*0.768*	*0.401*	*0.751*	*0.777*	*0.392*	*1*

Table 2. Pearson r Correlations for Predictor and Outcome Variables

The CC begins by subdividing the entire Pearson r correlation matrix into four sub-matrices: R**yy**, R**xx**, R**xy**, and R**yx**. **Ryy** is the matrix of correlations of outcome variables with themselves. **Rxx** is the matrix of correlations of predictors with themselves. **Rxy** and **Ryx** are redundant matrices of correlations among the predictors with the outcomes.

$\mathbf{Ryy}^{-1}$ **and** $\mathbf{Rxx}^{-1}$ are the inverses of the **Ryy** and **Rxx** submatrices. The computations yield several eigenvalues that derive from the solution of the determinant equation: $|R\mathbf{yy}^{-1}\ R\mathbf{yx}\ \ R\mathbf{xx}^{-1}\ R\mathbf{xy} - \square\mathbf{I}| = 0$. (Pedhauser, (1982, pps, 722-748) which are transformed into CC-Rs. Each eigenvalue has a corresponding set of raw weights that can be applied to the individual predictor and outcome data that maximizes the size of the CC-R. The CC also provides standardized regression weights ("□") and "structure weights" (SWP and SWO in Table 4) that summarize the relationship between each predictor and outcome with the variables in the opposite set. Each structure weight is a Pearson r coefficient that can be tested for significance as such.

The data structure for this sample CC included sets of four predictors and four outcomes. The predictor set included HSGPA, SATV, SATM, DEXT. The outcome set included GPAF, GPAS, Typing Speed, and Salary. The results of the CC are presented in Table 4 below. The analysis of these data yielded a CC-R of .997 which was significant, Wilk's Lambda = .000, F (16, 6.748) = 6.748, p < .05). All of the individual structure weights for the predictors (SWP in Table 4) were significant as were all of the structure weights for the outcomes (SWO in Table 4).

The size of the structure weights for the predictors indicates that HSGPA and SATV were the best (largest structure weights) predictors whereas the GPAF, GPAS and SALARY were the most sensitive outcomes.

The proposed analysis

Parente (1991) described how the MP could be used as an alternative to the CC in situations where the data are skewed or the sample sizes are small (Michaeli, Wang, & Livescu, (2016). The analysis involves applying the MP to a portion of a nonparametric (Spearman Rho) correlation matrix (upper right bolded values in Table 3). A major computational difference between the CC and MP approaches is that the former is based on the entire matrix of *Pearson r* correlations (Table 2) whereas the later uses only the submatrix of *Spearman Rho* correlations that summarize the relationship between the predictors and the outcomes.

The mechanics of MP analysis

The purpose and intent of the proposed MP is the same as it is for the CC. Both analyses assess multivariate relationships that exist between a set of predictors and a set of outcomes. However, the mechanics of the analyses differ markedly. The MP computations yield a "Common Value" (MP-CV) which is close to the median of all the values in the submatrix. The MP-CV and the CC-R measure the extent to which the predictor and outcome sets are related. Whereas the CC produces structure weights, the MP computes median row values (MedR) which represents the magnitude of relationship for each predictor across all of the outcomes. The MP also provides column medians (MedC) that represent the sensitivity of each individual outcome across the set of predictor variables. The analysis continues with a series of "sweeps" (Klawonn, et al, 2013) that yields residuals ("Effects") for each row or column median. These are measures of deviation of the predictor/outcome variables from the row or column values that contain them. The computational procedures for MP are tedious and are best left to reliable computer software (Velleman & Hoaglin, 1981). However, Klawonn (2013) outlined the basic steps involved:

1. "For each row, compute the row median, store it as a row median, and subtract it from the values in the corresponding row.
2. The median of the row medians is then added to the overall effect and subtracted from each row median.
3. For each column compute the median, store it as the column median and subtract it from the values in the corresponding column.
4. The median of the column medians is then added to the overall effect and subtracted from the column medians
5. Repeat steps 1-4 until no changes (or very small changes) occur in the row and column medians."

	HSGPA	**SATV**	**SAT,**	**DEXT**	**GPAF**	**GPAS**	**TYPE**	**SAL**
HSGPA	1.000	.976**	.939**	.321	***.939****	***.988****	***.236***	***.830****
SATV	.976	1.000	.891	.370	***.952****	***.988****	***.309***	***.782****
SATM	.939**	.891**	1.000	.406	***.927****	***.915****	***.309***	***.758****
DEXT	.321	.370	.406	1.000	***.467***	***.358***	***.964****	***.394***
GPAF	.939**	.952**	.927**	.467	1.000	.964**	.406	.685*
GPAS	.988**	.988**	.915**	.358	.964**	1.000	.273	.794**
TYPS	.236	.309	.309	.964**	.406	.273	1.000	.309
SALRY	.830**	.782**	.758*	.394	.685*	.794**	.309	1.000

Table 3. Spearman Correlation Matrix (Bolded Values in Upper Right) for Predictors and Outcome Measures

	GPAF	GPAS	Typing	Salary	MdnR	Effect	SWP
HSGPA	*.939*	*.988*	*. 236*	*.830*	**.748**	**.019**	**.885**
SATV	*.952*	*.988*	*.309*	*.782*	**.867**	**.010**	**.887**
SATM	*.927*	*.915*	*.309*	*.785*	**.850**	**-.010**	**.786**
DEXT	*.467*	*.358*	*.964*	*.394*	**.428**	**-.435**	**.737**
MdnC	**.933**	**.952**	**.309**	**.783**			
Effect	**.063**	**.082**	**-.556**	**-.062**			**Canonical R = .997 (p<.05)**
SWO (p<.05)	**.902**	**.851**	**.747**	**.817**			**Common Value = .873**

Table 4. Summary Statistics for CC and MP Analyses

Note: Med-R values index effect of predictors on outcomes. Med-C values assess the sensitivity of outcomes across predictors. SWP and SWC values are structure weights taken from CC. MP-CV is the end product of the MP after the sweeping process.

The MP-CV) is the end product of the sweeping process outlined above. This statistic can be tested for significance as a Spearman Rho and it's size is an index of the overall relationship between the predictor and the outcome variable sets. The MdnR and MdnC values, which are computed *before* the sweeping process begins, measure the extent to which the row or column variables contribute to the overall canonical relationship. The MP also provides "Effect" values which are the residuals computed on the row or column medians *after* the sweeping process. Large absolute Effect values suggest that the variable is an outlier relative to the overall set of predictors or outcomes.

RESULTS AND FINDINGS

Both the MP-CV and the CC-R represents the overall correlation between a set of predictors (rows) and the outcomes (columns); The CC-R is tested using the Wilks' Lambda statistic presented above. Assuming the CC-R test is significant, then it is reasonable to interpret the structure weights. The size of the structure weights (SWP and SWO, see Table 4) indicates the extent to which that predictor or outcome variable contributes to the overall canonical relationship between the sets. Specifically, structure weights are Pearson correlations between the individual predictor or outcome measures and the weighted linear combination of the variables in the opposite set. Regarding the present data, the SWP values in Table 4 are the structure weights for the predictors taken from the CC analysis. The SWO values are the structure weights for the outcomes. All of the predictor and outcome structure weights are statistically significant. However, the relative size of the structure weights suggests that the HSGPA and SATV are the best predictors; the GPAF, GPAS and SALARY variables are the most sensitive outcomes.

Because the MP-CV, MdnR, and MdnC are median Spearman Rho correlations, they can be tested for significance using conventional criteria. In this example, the MP-CV of .873 was significant (p < .050) which indicates an overall significant relationship between the predictor and outcome variable sets. The MedR and MedC values are computed *before* any sweeping operation is undertaken. Larger MdnR values indicate stronger relationships among the predictor and outcome sets. For example, regarding the bolded values in the rows of Table 4, SATV and SATM have the largest significant median MdnR values (.867, .850, p<.05). Dexterity produces the lowest median correlation across the outcomes. (.428, n.s.). Regarding the columns, GPAF and GPAS display the highest MdnC values across the predictor set (.952, .933, p<.05) and Typing Speed produced the lowest median correlation (.309, n.s.). Inspection of the row Effects indicates that the DEXT variable is an outlier. The remaining variables are relatively homogeneous. Regarding the column medians, the Effects indicate that GPAF and GPAS and Salary have similar medians whereas the TYPING variable is an outlier.

In general, the MP-CV value identifies an overall significant relationship between the predictor and outcome sets. The best predictors of college success and salary are performance on the Verbal, and Quantitative portions of the SAT. The most sensitive outcomes are Freshman and Senior GPA and Salary.

Monte Carlo Evaluation of CC and MP Procedures

Given, then, that the MP and CC approaches share a parallel application, the problem for researchers is to determine which of the two statistical analyses is more robust when parametric assumptions (e.g. skewness) are not met or the sample size does not ensure adequate power. The MP analysis should be more resistant to the effects of skewness than is the CC because nonparametric techniques are based on methods that minimize the effect of extreme values in a distribution (Zach, 2022). On the other hand, a long-standing assumption has been that nonparametric procedures may not have the same level of power relative to their parametric counterparts (Kiplinger, 1984; Mumby, (2002). It is therefore reasonable to assume that an analysis based on parametric assumptions such as the CC, may be unreliable when sample sizes are small or the distributions are skewed. Therefore, the purpose of this study was to determine the relative robustness of the CC-R statistic verses the MP-CV statistic with manipulations of skewness and sample size.

Wikipedia (https://en.wikipedia.org/wiki/Monte_Carlo_method) describes methods for comparing alternative statistical procedures in terms of “robustness”. The procedure involves assessing the proportion of times alternative statistical measures provide test statistics that fall outside the 95% confidence intervals, that is, for example, when the raw data are sample distributions derived from computer generated normal versus skewed distributions. Therefore, we compared the CC-R and

MP-CV statistics computed on normally distributed versus skewed distributions in each of four sample size conditions (n = 20, 30, 50, and 100).

Our Monte Carlo procedure first generated normally distributed data followed by a manipulation that skewed these same data distributions. The program computed the CC-R and MP-CV analyses on each of these data sets. The first simulation assessed the Type 1 error rate for the MP-CV and the CC-R statistics computed on normally distributed versus skewed data sets without regard to sample size. The second simulation compared the frequency of Type 1 errors for the CC-R and MP-CV analyses in each sample size conditions (n = 20, 30, 50, 100) without regard to skewness.

Hypotheses

The first hypothesis was that relative to the CC-R, the MP-CV procedure would provide a more robust estimate (fewer Type I errors) when the underlying distributions were skewed. The second hypothesis was that the MP procedure would produce fewer Type I errors relative to the CC approach when the sample sizes were smaller.

The computer program used in this study was originally published by Velleman and Hoaglin (1981, pps 244-248). The Monte Carlo methods used here involved computing random number series to produce data for the subgroups being studied. The software provided a random number generator function seeded by the number of seconds elapsed since midnight on the day the program was run. The randomness of the generated number strings was checked using the method of autocorrelation. That is, a typical random number string of 1000 values was auto-correlated at lags from 1, 10 and 100. The results of the test showed that none of the autocorrelations were significant. This result indicates that the computer's random number function produced strings of random numbers with uncorrelated error terms. In order to prevent a systematic bias that might occur from analyzing the MP and the CC with two different sets of Monte Carlo data strings, the CC and MP subroutines were written to perform parallel analyses based on the *same sets* of randomly generated raw data. In addition, the numbers in the strings for the simulations were not only randomly sequenced but were also normally distributed McNitt (1983). One thousand CC-R and the MP-CV statistics were computed in each of the skew and sample size conditions.

Skewing was achieved by multiplying raw data between one and two positive standard deviations, from the mean of the raw data column, by a factor of 1.5. This resulted in data distributions that were positively skewed. In terms of percentiles, the raw data between the 84th and 97.5th percentiles of the distribution of raw data was multiplied by a factor of 1.5. This resulted in a concentration of smaller scores occurring on the left side of the distribution. Test runs of samples of 20 observations showed that, under these conditions, the skewness index ranged from 1.28 to 2.25. The standard error of skewness for samples of 20 observations in this study was .5477. The effect of varying sample size was achieved by comparing the CC-R and MP-CV distributions with 20, 30, 50, or 100 observations.

RESULTS AND FINDINGS

Table 5 presents the data for the skewed versus normally distributed data *without regard to the sample size.*

	Canonical Correlation	Median Polish Analysis
Skewed Data Frequency	320/8%	240/6%
Normally Distributed Data	200/5%	200/5%

Table 5. Number and Percentage of Type 1 Errors from CC versus MP Analyses Summed Across Four Sample Size Conditions with Skewed data relative to Normally Distributed Data. Note: Each condition (skewed vs normal) contained 4000 replications.

Table 5 displays the frequency of aberrant Type I errors for the CC-R and MP-CV analysis based on 95% confidence limits of normally distributed data for each analysis. Analysis of the normally distributed data, revealed 200 Type 1 errors (5%). The percentage frequency of Type 1 errors for the skewed data using the CC-R statistic equaled 8% (320 errors) which was significantly larger than one would expect given the results of the same analysis using normally distributed data (□□= 72, p < .05). The Type I errors for the MP-CV analysis computed on the same skewed distributions was 6% (240 errors, □□□= 8, p<.05). Therefore, with skewed data, both analyses produced more Type 1 errors than would be expected with normally distributed data. However, this result is partially due to the inclusion of error due to variations in sample size which is discussed below. The CC procedure produced significantly more Type 1 errors relative to the error rate for the MP-CV data (320 vs 240, □□= 20, p < .05). This result provides support for our first hypothesis.

Table 6 illustrates the effect of group size on the frequency of Type I errors for different sample sizes *without regard to skewness.* Each of the sample size conditions was based on 1000 replications. The expected Type 1 error for each condition was 50 (.05 x 1000) which was consistent with the output of the computer software.

	Sample Sizes			
	20	**30**	**50**	**100**
CC- R	110/11%	78/7.8%	47/4.7%	58/5.8%
MP- CV	49/4.9%	39/3.9%	55/5.5%	60/6%
Normally Distributed Data	50/5%	50/5%	50/5%	50/5%

Table 6. Absolute and Percent of Type 1 Errors from CC-R and MP-CV analyses at Four Sampling Sizes.
Note: Expected frequency is 50 = 5% of 1000 normally distributed data for each sampling size.

The entries in Table 6 illustrate significant overall Type 1 error differences between all 4 sample size conditions relative to the expected values (χ^2= 89.14, $p < .05$) for the CC-R and but not for the MP-CV. The largest Type 1 error differences between the CC-R data versus the normally distributed data occurred in the 20 and 30 sample size conditions (χ^2= 87.68, $p < .05$) but not for the MP-CV variable. In these two conditions, the percentage of Type 1 errors for the MP-CV statistic was roughly half that for the CC-R (χ^2= 75.94, $p < .05$). There were generally no differences between Type 1 errors for the two analyses with larger sample sizes (50-100). These findings confirm our second hypothesis.

DISCUSSION

The CC and MP analyses share a similar intent and purpose. Both procedures assess the overall relationship between a set of predictor variables and a set of outcomes. CC uses the canonical R to assess this predictor/outcome overall relationship whereas the MP uses the common value CV. Both of these analyses assess which predictors and outcomes best define the overall relationship. The CC computes the correlation of each predictor or outcome with the corresponding weighted canonical variate from the opposite set (structure weight). The MP uses the median Spearman Rho computed across the rows or columns of the correlation table as an index of utility for the predictors and the sensitivity of the outcomes. Both CC and MP methods can be used for hypothesis testing or for exploratory research. Both analyses produce similar levels of Type I errors with sample sizes that are large (50-100).

There are, however, clear differences between these analyses. The CC-R is a parametric statistic that is bounded by assumptions whereas the MP-CV is a non-parametric procedure with relatively few assumptions. The CC requires large sample sizes and bell-shaped distributions whereas the MP does not require either. The CC analyzes the entire correlation matrix of Pearson r statistics whereas the MP analysis derives from a subset of non-parametric correlations that summarize the relationship between the predictors and outcomes (see Table 4 italicized correlations). The CC uses structure weights to assess the effect that the predictors have on the outcomes and the sensitivity of the outcomes with the predictors. The MP provides a similar analysis by testing the significance of the row and column Rho medians. The MP produces Effect values which are residuals that allow the researcher to investigate which variables are outliers. Those variables with small or non-significant row or column median Rho values and large Effects are outliers and can be eliminated from the overall model.

Implication to Research and Practice

The MP analyses may be especially useful for exploratory research where the goal is to identify significant multivariate relationships among large sets of variables in a data structure. Parsimonious models can be created by eliminating predictors or outcomes with nonsignificant MdnC or MdnR values that also have large Effect scores. A-priori hypothesis tests can be achieved by specifying a significant overall relationship among the predictors and outcomes and testing the corresponding MP-CV for significance. Individual MdR and MdC values can be tested as bivariate Spearman Rho statistics. The computation example used here was based on correlations which were all positive. However, it is likely that with large numbers of predictors or outcomes, at least some of the signed values will be negative. In this situation we recommend using squared values for the initial analysis followed by an individual interpretation of the Mdn-R or Mdn-C signed values.

CONCLUSION

The three major findings in these data are 1. The MP produces fewer Type I errors relative to the CC when the sample data are skewed. 2. The CC's Type 1 error rate increases as the sample size decreases whereas the MP Type 1 error rate remains stable relative to the CC with changes in sample size. These finding are consistent with Rasmussen and Dunlap (1991) for non-normally distributed raw data. 3. In general, these results suggest that the MP is the better choice for analyzing canonical relationships when the sample sizes are less than 50 and when the data are significantly skewed.

Future Research

Replications of these findings using sample data from existing experiments would help to validate the conclusions presented above. The experimental design reported here restricted the number of predictors and outcomes to four each. However, the MP and the CC can be extended to any number of predictors or outcomes. Ideally, the Monte Carlo portion of this study could be extended to include more variables and combinations of different numbers of predictors and outcomes. Although these data suggest that the MP is the better choice of analytic procedure with small samples, additional Monte Carlo modeling should investigate Type 1 error rate for the CC-R and MP-CV statistics in situations where the number of cases approached the number of variables. It would be especially useful for exploratory researchers to develop the MP as a stepwise analysis that would drop off non-significant predictors and outcomes with large Effect values thereby creating a parsimonious canonical models that would include only those predictors and outcomes that contribute to the overall canonical relationship. Although we focused this paper on the effects of skewness and sample size on the global CC-R and MP-CV statistics, future studies should assess the effect of skewness and sample size on the row and column medians of the MP analysis.

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