Numerical Quantification of Co-Existence and Survival of White Yam and Yellow Yam Species: Variations of Intra-Species Coefficients

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Citation: Ogoegbulem O., Nwala B O., and Obukohwo V. (2022) Numerical Quantification of Co-Existence and Survival of White Yam and Yellow Yam Species: Variations of Intra-Species Coefficients, *International Journal of Physical Sciences Research*, Vol.6, No.1, pp.1-14

ABSTRACT: *In this study, utilizing a computationally effective numerical approach called ODE45, the impacts of decreased and increased variability of intra-species coefficients combined to some extent on coexistence and survival scenarios were successfully studied. In spite of diminished or enhanced variability, the two biological species co-exist dominantly, according to the study's numerical analysis. When all parameter values were fixed and the intra-species coefficients were reduced, the findings appeared to show that only white yam could coexist and survive. However, the reduced variation merely favoured the carrying capacity values and the unfavourable coexistence and survival scenario. The two species' ability to coexist and survive in this environment was largely favoured by increasing variability, although carrying capacity estimates were negatively impacted. We also discovered that, as the intra-species coefficient variation rose, both biological species were able to coexist and survive dominantly. The complete findings that we have achieved were shown in the tables after meeting both coexistence and survival criteria.*

KEYWORDS: Co-existence, survival, variation, intra-species coefficients, species, numerical, quantification

INTRODUCTION

The coexistence and survival of biological species is a long-standing scientific topic, however there is currently no quantitative and statistical data to support it (Ekaka-a et al., 2018 & Nwala, 2020). In this context, nearly all scientists, and particularly biologists, are interested in identifying and predicting the types of interactions that biological species go through to improve coexistence and survival in order to ensure optimal reproduction. This is a crucial issue that requires immediate national and international intervention (Nu-ue and George, 2021, Dubey and Hussain, 2006). According to research, coexistence is the design that fosters the ecological community, and the development of ecosystems is hampered by the spatial distribution of the surrounding species and their competition for the few resources that are available (Naam et al., 2021; Ekaka-a, 2013 & Seifert and Seifert, 2014).

The use of the numerical quantification technique is a specified mathematical idea that powers a model of biological species competition using ordinary differential equations that is consistent with the works of (Ekaka-a et al., 2018; Ford et al., 2010; Nafo and Ekaka-a, 2013 &Nwala, 2020). Additionally, the system of ordinary differential equations considered in this study is nonlinear and lacks a closed form solution, making it difficult and rare to find an analytical solution. To get around this issue, a numerical simulation was investigated as an alternative approach to determine the effects of decreased and increased variations of intra-species coefficients on the coexistence and survival of two species (Ekaka-a et al., 2018 & Nwala and Nwagor, 2021).

This crucial mathematical principle is a necessary tool for research on ecosystem management, application, and functioning, respectively. The proactive understanding of coexistence is not merely a solution to an intellectual conundrum; rather, it may improve to redress managerial concerns like the conservation of rare species and a cornerstone of coexistence theory (Decesare et al., 2010 & Levine and Hart, 2017).Inter-species interactions are those between two distinct species, whereas intra-species interactions are those between two different species in this context (Bengtsson, 1989; Ekaka-a, 2013 & Abranum et al., 2020).Depending on the mathematical structures that define them, these two ideas, referred to as intra-species and inter-species coefficients, contribute either favourably or unfavourably to the intrinsic growth rates.

MATHEMATICAL FORMULATIONS

We have taken into consideration a continuous dynamical system of nonlinear first order ordinary differential equations that, according to Nwala, describes the interactions between two biological species (2020).

$$\frac{dY_1(t)}{dt} = \propto_1 Y_1(t) - \beta_1 Y_1^2(t) - K_1 Y_1(t) Y_2(t)$$

$$\frac{dY_2(t)}{dt} = \propto_2 Y_2(t) - \beta_2 Y_2^2(t) - K_2 Y_1(t) Y_2(t)$$

With initial conditions

$Y_1(0) = Y_1(0) > 0$, $Y_2(0) = Y_2 > 0$

<u>Positivity and Boundedness of Solutions</u>

By demonstrating the positivity, boundedness, existence, and uniqueness of the model's solutions with non-negative beginning data, we can show that the crops growth model is well-posed and that non-negative data remains constant for all time t > 0, which is the key to our goal.

LEMMA 1: The set $\Omega = \left\{(Y_1, Y_2) \epsilon R_t^2 : N \leq \frac{\alpha_1}{\beta_1}\right\}$ is positively invariant with respect to the growth model.

LEMMA 2: Positivity: Let the set of initial data be $\{(Y_1(0), Y_2(0)) \epsilon \emptyset\}$; then the dataset$\{Y_1(t), Y_2(t)\}$ of the growth model is non-negative for $t > 0$.

Proof: Assuming that the set of initial data, $\{(Y_1(0), Y_2(0)) \geq 0\}$ then

$$\frac{dY_1(t)}{dt} = \alpha_1 Y_1(t) - \beta_1 Y_1^2(t) - K_1 Y_1(t) Y_2(t)$$

$$\frac{dY_1}{dt} = Y_1(t)\,(\alpha_1 - \beta_1 Y_1 - K_1 Y_2)$$
$$\frac{dY_1}{Y_1} = (\alpha_1 - \beta_1 Y_1 - K_1 Y_2)dt$$
$$\int \frac{dY_1}{Y_1} \geq \int (\alpha_1 - \beta_1 - K_1 Y_2)dt$$
$$Y_1(t) \geq Y_1(0)e^{(\alpha_1 - \beta_1 Y_1 - K_1 Y_2)} \geq 0$$

Similarly, $Y_2(t) \geq Y_2(0)e^{(\alpha_2 - \beta_2 Y_2 - K_2 Y_1)} \geq 0$

LEMMA 3: Boundedness; Assuming that the initial condition for the growth model satisfies $N(0) \leq \frac{\alpha_1}{\beta_1}$, Thus wherever the solution exists on an interval I the property of boundedness is satisfied. $N(t) \leq \frac{\alpha_1}{\beta_1}$.

Proof: Since $I(t) \geq 0$,

$$\frac{dN}{dt} < \frac{\alpha_1}{\beta_1}$$
$$N(t) \leq \frac{\alpha_1}{\beta_1} + N(0) - \left(\frac{\alpha_1}{\beta_1}\right) e^{\beta_1 t}$$
$$where\ N(0) \leq \frac{\alpha_1}{\beta_1}$$

We have, $N(t) \leq \frac{\alpha_1}{\beta_1}$

LEMMA 5: Existence and Uniqueness

Let $t > 0$, if the initial data satisfies $\{(Y_1(0), Y_2(0)) > 0\}$then $\forall\, t \in R$, the solution set $\{(Y_1(t), Y_2(t))\}$ exists inR_t^2 and is unique.

Proof: The growth model could be written as

$$x = \begin{vmatrix} Y_1(t) \\ Y_2(t) \end{vmatrix}\ and\ F(x) = \begin{vmatrix} Y_1\,(\alpha_1 - \beta_1 Y_1 - K_1 Y_2) \\ Y_2\,(\alpha_2 - \beta_2 Y_2 - K_2 Y_1) \end{vmatrix}$$

Thus, F is continuous and locally satisfies Lipschitz condition in R_t^2 using the Picard-Lindel öf theorem.

The first and second lemmas showed the positivity and boundedness of the solution, there exists $t > 0$ such that the result set of the growth model exists locally at least on the interval $[0, t]$ and is unique.

Therefore, lemma 4 satisfied the production validity of the growth model.

LEMMA 5: Therefore, satisfied that the growth model is well-posed and valid in the set

$$\int \Omega = \int (Y_1(t), Y_2(t)) \in R_t^2 : N \leq \frac{\alpha_1}{\beta_1}$$

Where,
$Y_1(t)$ represents the biomass of white yam species at time t in the unit of months.
$Y_2(t)$ represents the biomass of yellow yam species at time t in the unit of months.
$\propto_1$ and $\propto_2$ represents the growth rates of the two yam species known as white and yellow yam species.

β_1 and β_2 represents the intra-species coefficients of white and yellow yam species respectively.
K_1 and K_2 represents the inter-species coefficients between the white and yellow yam species respectively.

In this study, the following parameter values have been considered according to Nwala&Nwagor (2021)
$\propto_1 = 0.1, \propto_2 = 0.08, \beta_1 = 0.0014, \beta_2 = 0.001, K_1 = 0.00012 \; and \; K_2 = 0.0009$

MATERIAL AND METHOD

We used a MATLAB ODE45 numerical approach to solve the dynamical system of nonlinear first order ordinary differential equations in order to effectively solve and analyse the ecological issue of coexistence and survival in the framework of two biological crops contending for scarce available resources.

CALCULATION OF CARRYING CAPACITY

Carrying capacity, when used to represent a real-world issue, refers to the largest population size that can support population growth (Nwala and Nwagor, 2021; George, 2019 & Nwala, 2022).In mathematics, carrying capacity is determined by dividing the growth rate by the intraspecies coefficient. Thus,

Carrying capacity $CC(Y_1) = \frac{\propto_1}{\beta_1} = \frac{0.1}{0.0014} = 71.4286 \cong 71.43g/A$
Carrying capacity $CC(Y_2) = \frac{\propto_2}{\beta_2} = \frac{0.08}{0.001} = 80.00g/A$

To the best of my knowledge, Ford et al. (2010) and Nwala (2020) claimed that the species with the highest value of carrying capacity stands a greater chance to resist against the vulnerability to environmental perturbation than the species with low value of carrying capacity. According to the estimate, the yellow yam (Y2) has a greater chance than the white yam to disrupt environmental perturbation (Y1).
In agreement with Naam et al., (2021), who claimed that the inequalities signs must be met for two species to survive, that is:

(1) $\propto_{12} = \frac{K_1}{\beta_1} < \frac{CC(Y_1)}{CC(Y_2)}$

(2) $\propto_{21} = \frac{K_2}{\beta_2} < \frac{CC(Y_2)}{CC(Y_1)}$

Where

$$\propto_{12} = \frac{K_1}{\beta_1} = \frac{0.00012}{0.0014} = 0.0857$$

Similarly,

$$\propto_{21} = \frac{K_2}{\beta_2} = \frac{0.0009}{0.001} = 0.9000$$

Thus, $\frac{CC(Y_1)}{CC(Y_2)} = \frac{71.43}{80.00} = 0.8929$

Similarly,

$$\frac{CC(Y_2)}{CC(Y_1)} = \frac{80.00}{71.43} = 1.1200$$

Consequently, CC(Y1) is the carrying capacity of the white yam, CC(Y2) is the carrying capacity of the yellow yam, and _12 defines the contribution of the yellow yam to limit the growth of the white yam, whereas _21 describes the contribution of the white yam to prevent the growth of the yellow yam. Therefore, if the two aforementioned conditions are met, the two perennial crops will coexist and thrive together in the same habitat.

PRESENTATION OF RESULTS

The results of this study are displaced in the tables below.

Table 1:

Month LGS	β_1	β_2	CC(Y_1)	CC(Y_2)	$\propto_{12}$	$\frac{CC(Y_1)}{CC(Y_2)}$	$\propto_{21}$	$\frac{CC(Y_2)}{CC(Y_1)}$	Decreasing variations of β_1 and β_2
1	0.0014	0.0010	71.43	80.00	0.0857	0.8929	0.9000	1.1200	β_1 = 0.0014 β_2 = 0.001
2	0.00014	0.0001	7114.28	800.00	0.8571	0.8929	9.0000	1.1200	10%
3	0.00022	0.00016	476.18	533.33	0.5455	0.8929	5.6250	1.1200	15%
4	0.00027	0.00020	357.13	400.00	0.4444	0.8929	4.5000	1.1200	20%
5	0.00036	0.00026	285.70	320.01	0.3333	0.8929	3.4615	1.1200	25%
6	0.00041	0.00031	238.11	266.66	0.2927	0.8929	2.9032	1.1200	30%
7	0.00048	0.00036	204.09	288.56	0.2500	0.8929	2.5000	1.1200	35%
8	0.00057	0.00040	178.58	200.00	0.2105	0.8929	2.2500	1.1200	40%
9	0.00064	0.00046	158.74	177.77	0.1875	0.8929	1.9565	1.1200	45%
10	0.00070	0.00051	142.88	160.00	0.1714	0.8929	1.7647	1.1200	50%

Evaluating the effect of decreasing intra-species coefficients together on the co-existence and survival scenario of white yam and yellow yam using ODE45 numerical method.

Month LGS	β_1	β_2	CC(Y_1)	CC(Y_2)	$\propto_{12}$	$\frac{CC(Y_1)}{CC(Y_2)}$	$\propto_{21}$	$\frac{CC(Y_2)}{CC(Y_1)}$	Decreasing variations of β_1 and β_2
1	0.0014	0.001	71.43	80.00	0.0857	0.8929	0.9000	1.1200	β_1 = 0.0014 β_2 = 0.001
2	0.00078	0.00056	129.88	145.46	0.1539	0.8929	1.6071	1.1200	55%
3	0.00083	0.00060	119.06	133.33	0.1446	0.8929	1.5000	1.1200	60%
4	0.00092	0.00066	109.88	123.07	0.1304	0.8929	1.3636	1.1200	65%
5	0.00097	0.00070	102.05	114.28	0.1237	0.8929	1.2851	1.1200	70%
6	0.00105	0.0076	95.25	106.66	0.1143	0.8929	1.1842	1.1200	75%
7	0.00111	0.00080	89.28	100.00	0.1091	0.8929	1.1250	1.1200	80%
8	0.00118	0.00085	84.04	94.11	0.1017	0.8929	1.0588	1.1200	85%
9	0.00125	0.00090	79.36	88.89	0.0960	0.8929	1.0000	1.1200	90%
10	0.00132	0.00095	75.18	84.20	0.0909	0.8929	0.9474	1.1200	95%

Table II: Evaluating the effect of decreasing intra-species coefficients together on the co-existence and survival scenario of white yam and yellow yam using ODE45 numerical method

Month LGS	β_1	β_2	CC(Y_1)	CC(Y_2)	$\propto_{12}$	$\frac{CC(Y_1)}{CC(Y_2)}$	$\propto_{21}$	$\frac{CC(Y_2)}{CC(Y_1)}$	Increasing variations of β_1 and β_2
1	0.0014	0.001	71.43	80.00	0.0857	0.8929	0.9000	1.1200	β_1 = 0.0014 β_2 = 0.001
2	0.001414	0.001010	70.72	79.21	0.049	0.8929	0.8912	1.1200	101%
3	0.001425	0.001020	70.18	78.43	0.0842	0.8929	0.8823	1.1200	102%
4	0.001442	0.001030	69.35	77.67	0.0832	0.8929	0.8738	1.1200	103%
5	0.001456	0.001040	68.68	76.92	0.0824	0.8929	0.8654	1.1200	104%
6	0.001470	0.001050	68.03	76.19	0.0816	0.8929	0.8571	1.1200	105%
7	0.001484	0.001060	67.39	75.47	0.0809	0.8929	0.8491	1.1200	106%
8	0.001498	0.001070	66.76	74.77	0.0801	0.8929	0.8411	1.1200	107%
9	0.001512	0.001080	66.14	74.07	0.0794	0.8929	0.8333	1.1200	108%
10	0.001526	0.001090	65.53	73.40	0.0786	0.8929	0.8257	1.1200	109%

Table III: Evaluating the effect of increasing intra-species coefficients together on the co-existence and survival scenario of white yam and yellow yam using ODE45 numerical method

Month LGS	β_1	β_2	CC(Y_1)	CC(Y_2)	$\propto_{12}$	$\frac{CC(Y_1)}{CC(Y_2)}$	$\propto_{21}$	$\frac{CC(Y_2)}{CC(Y_1)}$	Increasing variations of β_1 and β_2
1	0.0014	0.001	71.43	80.00	0.0857	0.8929	0.9000	1.1200	β_1 = 0.0014 β_2 = 0.001
2	0.001541	0.001100	64.94	72.74	0.0779	0.8929	0.8181	1.1200	110%
3	0.001553	0.001110	64.36	72.06	0.0773	0.8929	0.8108	1.1200	111%
4	0.001567	0.001120	63.79	71.44	0.0766	0.8929	0.8036	1.1200	112%
5	0.001581	0.001130	63.20	70.80	0.0759	0.8929	0.7965	1.1200	113%
6	0.001597	0.001140	62.67	70.17	0.0751	0.8929	0.7895	1.1200	114%
7	0.001600	0.001150	62.10	69.55	0.0750	0.8929	0.7826	1.1200	115%
8	0.001623	0.001160	61.59	68.98	0.0739	0.8929	0.7759	1.1200	116%
9	0.001639	0.001170	61.06	68.39	0.0732	0.8929	0.7692	1.1200	117%
10	0.001653	0.001180	60.54	67.80	0.0726	0.8929	0.7627	1.1200	118%

Table IV: Evaluating the effect of increasing intra-species coefficients together on the co-existence and survival scenario of white yam and yellow yam using ODE45 numerical method

DISCUSSION OF RESULTS

In this paper we have Considered tableI, when every other parameter valueis fixed but, only the intra-species coefficients defined by β_1 and β_2 were varied by 10%, 15%, …, 50% decreased. This result identified that the two biological species co-exist and only white yam survived provided that $\propto_{12}$ is relatively less than $\frac{CC(Y_1)}{CC(Y_2)}$ from 10% variation to 50% variation approximately 0.8571 to 0.1714 respectively while the yellow yam does not survived hence the values of $\propto_{21}$ is greater than the values of $\frac{CC(Y_2)}{CC(Y_1)}$ ranging from 9.000 to 1.7647 respectively. The table II result established different behavior of biological species. The table captured co-existence of both crops from 55% to 95% respectively while there was evidence of white yam survival from 55% to 85% and thereafter, does not survive again from 90% to 95%. Moreover, the table also proved that the yellow yam does not survive within the range of 55% to 80% and suddenly survived from 85% to 95% provided the inequality sign is satisfied.

Looking at table III, which is denoted by the increment variations of β_1 and β_2 from 101% to 109% respectively was found that both biological crops relatively co-exist and dominantly survived provided that $\propto_{12}<\frac{CC(Y_1)}{CC(Y_2)}$ and $\propto_{21}<\frac{CC(Y_2)}{CC(Y_1)}$ which satisfied the survival criteria of ecological community. Similar trend is associated with table IV and V where the two crops co-exist and survived irrespective of the increased variations respectively.

The result of table IV, recorded a monotone decreased on the carrying capacity of both species from the value of 71.43kg to 60.54kg with respect to white yam and from the 80.00kg to 67.50kg with respect to yellow yam. Apparently, the result also indicated the satisfaction of co-existence and survival conditions of an ecological community provided that $\propto_{12}<\frac{CC(Y_1)}{CC(Y_2)}$and $\propto_{21}<\frac{CC(Y_2)}{CC(Y_1)}$ when the variation of β_1 and β_2 were increased from 110% to 118% respectively.

CONCLUSION

In the case study, we examined the numerical quantification of the co-existence and survival of the white yam and yellow yam species. It was determined that a decrease in the variation of the inter-species coefficients results in favourable carrying capacity values and is unfavourable to the co-existence and survival of the two competing crops, while an increase in the variation of the same parameter values results in unfavourable carrying capacity values and dominantly favours co-existence. Studying the impact of differences in inter-species coefficients on the coexistence and survival of two species can be done using the current technique to analysis.

Vol.6, No.1, pp.1-14, 2022
Print ISSN 2515-0391 (Print)
Online ISSN 2515-0405 (Online)

Appendix

Table 1: Evaluating the effect of decreasing intra-species coefficients together on the co-existence and survival scenario of white yam and yellow yam using ODE45 numerical method.

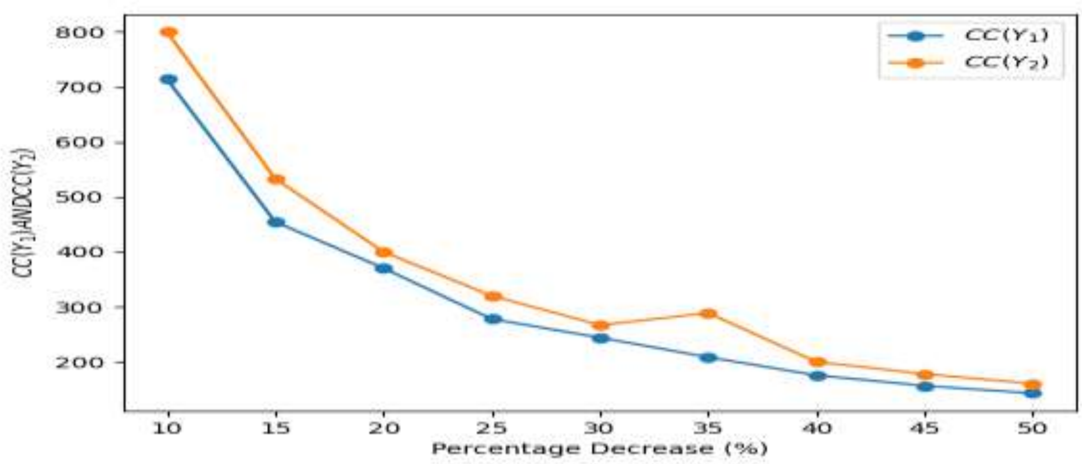


Table II: Evaluating the effect of decreasing intra-species coefficients together on the co-existence and survival scenario of white yam and yellow yam using ODE45 numerical method

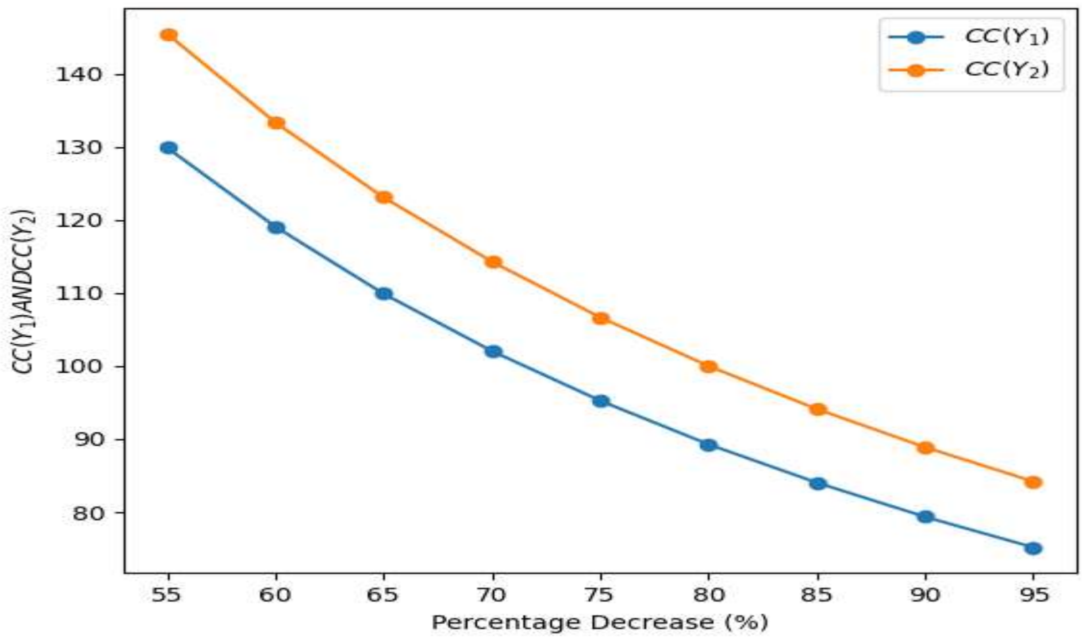


Table III: Evaluating the effect of increasing intra-species coefficients together on the co-existence and survival scenario of white yam and yellow yam using ODE45numerical method

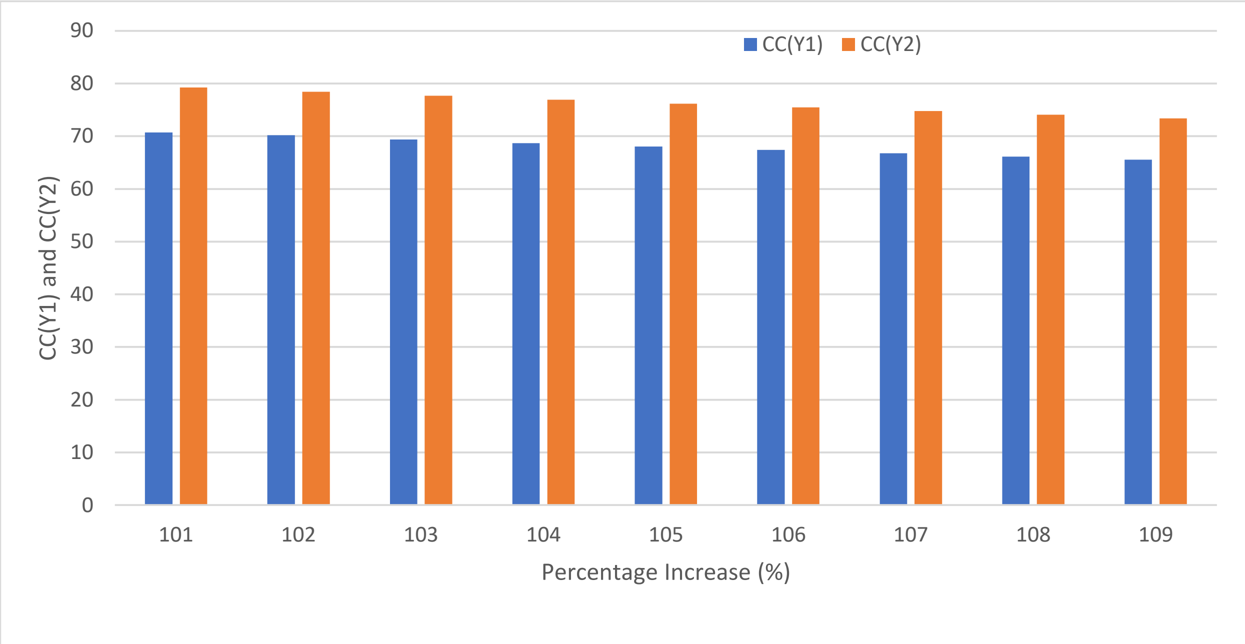
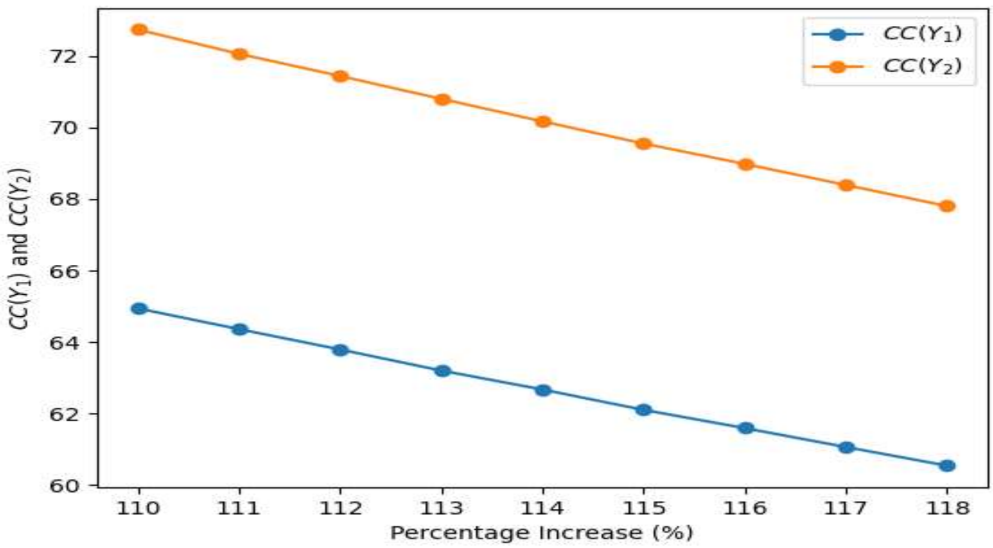


Table IV: Evaluating the effect of increasing intra-species coefficients together on the co-existence and survival scenario of white yam and yellow yam using ODE45 numerical method



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